Dr. PANTELIS DELIBALTAS

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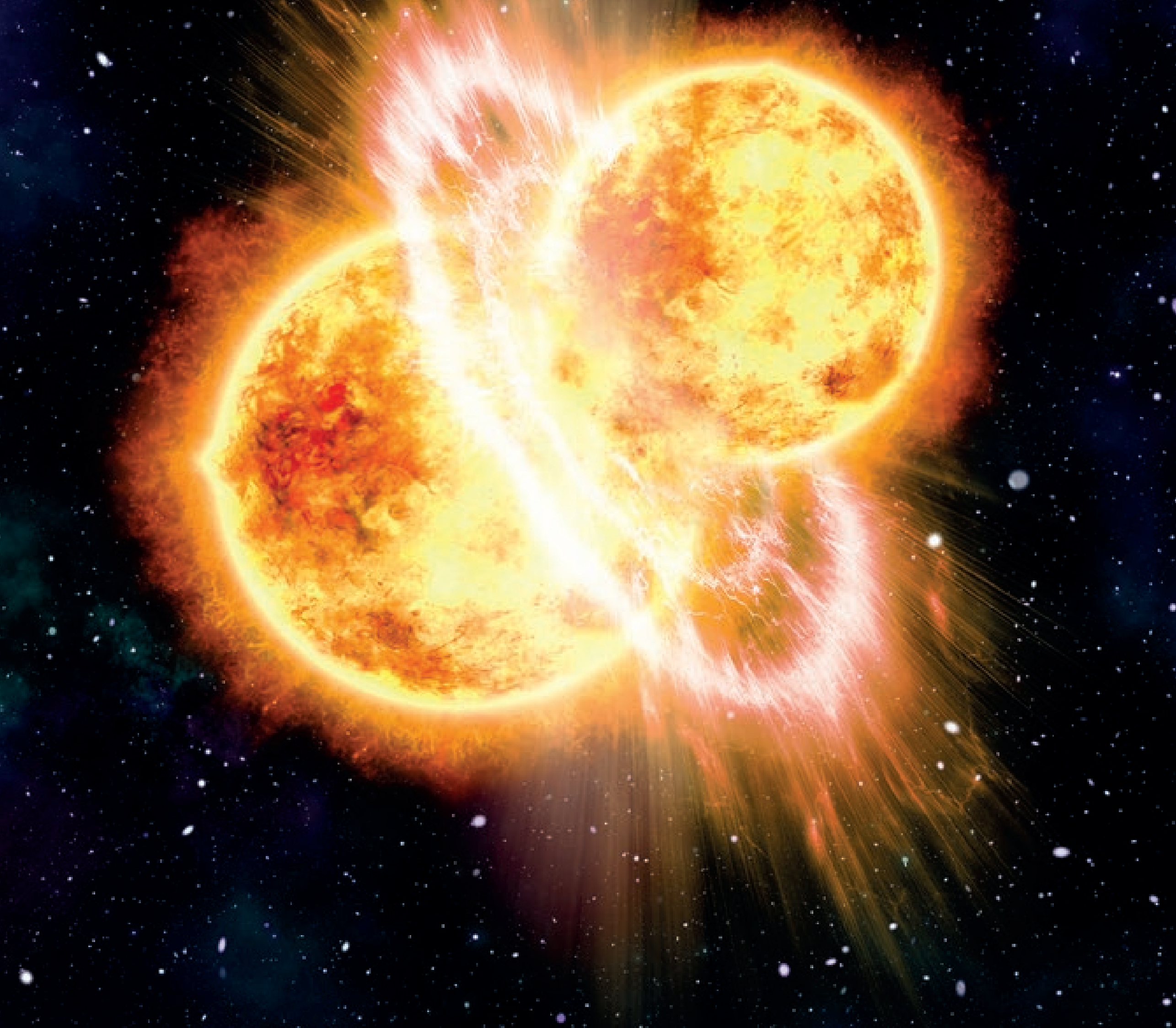
FROM THE THREE-BODY PROBLEM TO THE BIG BANG AND THE APPEARANCE OF LIGHT



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FROM THE THREE-BODY PROBLEM TO THE BIG-BANG AND THE APPEARANCE OF LIGHT

Dr Pantelis Delibaltas

Thessaloniki, 2021

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Collisions: From the Three-Body Problem
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PREFACE

During my study of physics at the University of Munich, I was very lucky to have attended lectures from the famous Prof. W. Heisenberg.

At the Technical University of Munich I attended lectures from the Nobel prize laureate Prof. R. Mössbauer. In the time of my Diploma studies I worked under Prof. N. Riehl at the nuclear reactor in Garching at Munich, about the diffusion of the O18 isotope in ice (P. Delibaltas et al). Under Prof. J. Chatzidimitriou and G. Bozis, I found periodic and periodic collision orbits in the general three body problem. With those studies I obtained the Doctor award in physics of the Aristotle University of Thessaloniki.

In this book I describe some events of the cosmology. I tried to make more accessible different difficult meanings of the cosmology, using a vacuum reference volume. I compared the pressure, the expansions work, the heat and the intrincing energy of the reference volume to the analogous observables of the universe.

I describe via collisions the birth of a planet's satellite and then I present a model for the Big-Bang.

In the following I circumscribe some of the latest research results of Physics at CERN in Geneva in a more comprehensible way and finally I describe some Cosmology observables.

From this side I want to express my thanks to Helena for her linguistic help and Evan for his technical help and scientific discusions.

P.E. Delibaltas

Thessaloniki, March 2021

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INTRODUCTION

We will describe three events that take place in our universe and try to explain them using physic's tools, in the same way, as the seismologists explain the earthquakes and the volcanologists explain the volcanoes. In the first case, of periodic elastic collisions, we made it with an accuracy to eight decimal places.

In the cases of inelastic and plastic collisions we did try to explain them qualitatively based on laws of physics and observations.

- Periodic Elastic Collisions with relatively small velocities in the three-body problem, (the Sun, Jupiter and Saturn) where both the energy and the total momentum are conserved.
- Plastic collision of two bodies with enormous velocities and masses producing temperatures of the order 10^{12}K, where the two bodies unite, their masses vaporize, leaving a vacuum behind them. In this way we will present in Sec. 8 a model for the Big-Bang. In this case the mechanical energy of the two bodies is not conserved because it is transformed into heat, whereas the momentum (locally and totally) is conserved. Further we will try to give a physical explanation to some strange meanings as the negative pressure and the dark energy.
- Inelastic collisions of two bodies with great velocities, where their total energy is partly transformed into heat, sound and intrincic deformation energy. Parts of their masses are hurled far away. The total momentum of the bodies is also not conserved.

Finally, we will describe some general issues of Cosmology and explain how the universe had been dark for about 380000 yrs. How at the beginning of the Big-Bang all particles were massless and how they received mass to build finally the Hydrogen-atom to send us light.

1. PERIODIC COLLISION ORBITS IN THE THREE-BODY PROBLEM

Four families of periodic collision orbits are published in an earlier work (P. Delibaltas, 1982). In one orbit of a family the three bodies consisted of the Sun, the Jupiter and the Saturn. The collision took place between the Sun and Saturn. The third body Jupiter stood far away, (J. Chatzidimitriou, G. Bozis). The initial conditions were the regularized at $t = 0$ infinite momenta by a (Waldvogel) transformation and the Sun and Saturn collided again after any period T. These orbits are calculated with accuracy to eight decimal places.

In an unit system, where the gravitational constant G=1 and the total mass $m_{SU} + m_{JU} + m_{SA} = m_{TO}$ was $m_{TO} = 1$ the conserved total momentum and mechanical energy were $p = 6.456 \times 10^{-4}$ and $en = -6.58 \times 10^{-4}$ respectively, in the above defined units, Fig.1.1.

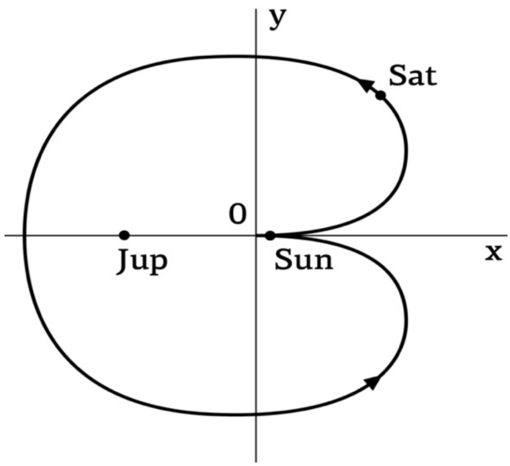

Fig. 1.1

To the question of some readers of the study, if one could numerically continue this orbit to a collision in a Big-Bang mode, the answer is NO because the mechanical energy in the case of the Big-Bang is not conserved.

2. PERFECT INELASTIC (PLASTIC) COLLISION BETWEEN TWO HOT BODIES

Let us assume that two hot bodies B_1 and B_2 are in the state of a plasma with nearly equal restmasses m_{i0} (without restriction of the generality and $i = 1, 2$). We assume that they have been found close to each other, their distance r very small perhaps because of pressure gradients, so that the repulsion because of the cosmological constant Λ which is added to the Einstein equations to stabilize the universe is negligible in relation to the gravitational attraction $\sim 1/r^2$ (see sect. 6.2) and it can be taken to be Λ=0. Thus the bodies B_1 and B_2 come to a collision. Let us divide the spheres in infinitesimal slices dm_i perpendicular to the axis B_1-B_2. After the collision they build an embodiment. The plasma is an ionized gas charged with ions and electrons giving rise to electric fields and charged particles flowing give rise to currents and magnetic fields, consequently to radiation, (R.J. Goldston, P.H. Rutherford). The energy density of a mass density ρ_M is transformed into radiation density energy $\rho_R = T^4$. We are interested in the motion of the bodies and the momenta along the axis B_1-B_2 and ignore the intrinsic linear velocities in other directions or rotations of the plasma ingredients, because of their high temperature.

- The bodies having an initial total momentum $\mathbf{p} = \mathbf{0}$, will conserve it and after the collision. The first collision takes place between the first infinitesimal slices $dm_1$and $dm_2$and the sum of the slices momenta $d\mathbf{p_1} + d\mathbf{p_2}$ along the B_1-B_2 axis will be zero, because of the conservation of the momentum (Fig. 2.1). At the collision the velocities of the slices become zero and the masses dm_i become restmasses dm_{i0}. Because of the infinitesimal thickness of the slices and the infinitesimal collision time between dm_1 and dm_2 the velocities of the bodies' constituents in other directions then the B_1-B_2 are unchanged.

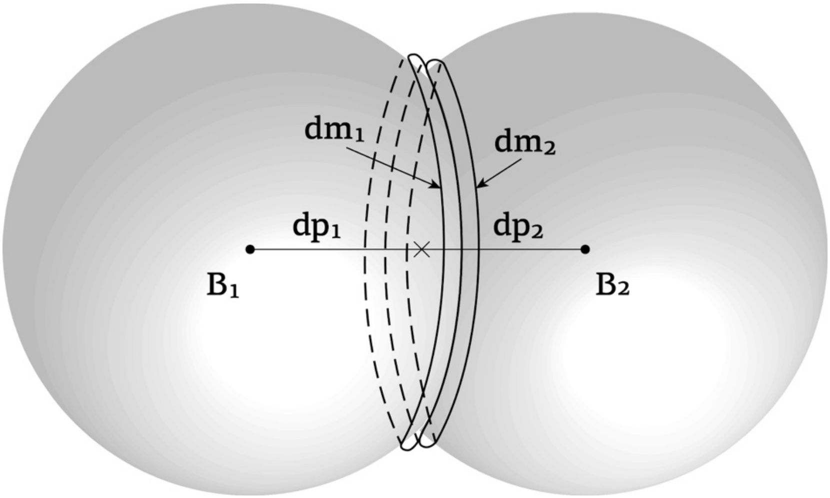

Fig. 2.1

The bodies do behave as solid. The mass-differences will be transformed into heat $dQ = dm_0(-1 + 1/\sqrt{1 - v_i^2})$ and afterwards into radiation with $v_i = 1 - 10^{-x}$ and $dm_0 = dm_{10} + dm_{20}$, increasing the temperature of the bodies B_1, B_2.The next sphere section will do the same, if we suppose

- at first approximation that the deceleration of the bodies is cancelled by their temperature increase and the increase of their energy density. Then we can assume that x is a constant from the begin to the end of the collision. We are expecting for x to be

$$1 \ll x \leq 6, \qquad (2.1)$$

being set for a reasonable velocity $v = 1 - 10^{-x}$. So neglecting the second order of 10^{-x} we integrate

$$Q_i = \int(-1 + 1/\sqrt{1 - v^2})dm_i = m_{i0} 10^{x/2}/\sqrt{2}\ \mathrm{GeV},$$

for each body.

The energy density of the two bodies is

$$2\rho_R \times 10^{x/2}\ \mathrm{GeV}^4 = \sqrt{2}T^4 \times 10^{x/2}\ \mathrm{GeV}^4 \qquad (2.2)$$

Because of the additional heat $Q = Q_1 + Q_2$ the plasma flies away, leaving behind a vacuum and a negative pressure p_V according to cosmologists (see Sect. 4). The volumina of the two bodies are $V_V = V_1 + V_2$. The expansion against the pressure $p \approx 0$ gives rise to produce a work

$$A = (p - p_V)[V - (V - V_V)] = -p_V V_V > 0,\ \ p_V < 0 \qquad (2.3)$$

$V - V_V$, V being the volume of the universe before and after the collision.

3. IMPERFECT INELASTIC COLLISION BETWEEN TWO COLD BODIES AND BIRTH OF SATELLITES

If two bodies B_1 and B_2 with restmasses m_{01} and m_{02}, solid surfaces and mass densities nearly equal, have been found close to each other, thus Λ can be set to zero and the gravitational force attracts and leads them to a lateral frontal collision .Let us assume that $\mathrm{m}_{01} < \mathrm{m}_{02}$ and that the impact is an imperfect inelastic collision .This means that the bodies lose energy and run separated after the impact. The total momentum of the two bodies is also not conserved.

The energy is transferred mainly into heat Q, sound and intrincing deformations energy. The velocities v_i and the restmasses $\mathrm{m}_{0\mathrm{i}}$ $(\mathrm{I} = 1, 2)$ grow up by gravitation attraction before the collision and at the collision the velocity of the body B_1changes direction, going through zero.

The velocity of the body B_2 decreases at the collision from v_2 to v_{02}.

The body B_1 starting from zero velocity moves behind the body B_2, which is starting with velocity $\mathrm{v}_{02} > 0$.

The masses grow up according to

$$\mathrm{m}_\mathrm{i} = \mathrm{m}_{0\mathrm{i}}/\sqrt{1 - \mathrm{v}_\mathrm{i}^2} \tag{3.1}$$

with the non-relativistic velocity $\mathrm{v}_\mathrm{i} \ll \mathrm{c}$ and $\mathrm{c} = 1$, the velocity of light. Thus the mass-differences at the collision are

$$\mathrm{m}_1 - \mathrm{m}_{01} = \mathrm{m}_{01}(-1 + 1/\sqrt{1 - \mathrm{v}_1^2}) \text{ and}$$

$$m_2 - m_{02} = m_{02}(1/\sqrt{1 - v_2^2} - 1/\sqrt{1 - v_{02}^2})$$

and are transformed, mainly into heat Q_i. If $Q_1 + Q_2$ is great enough some parts of the contact masses vaporize, fly far away and some time after the impact undercool, begin to condensate and build a soft body, orbiting around the nearest body B_1. Through collisions with the matter of the universe (Eq. 4.2) the soft new body takes the most stable sphere form. A satellite of B_1 has been created. The soft surfaces of the bodies B_1 and B_2 become mountains and canyons.

4.THE INTERPRETATION OF THE NEGATIVE PRESSURE AND THE EXPANSIONS WORK

If one asks what is the physical meaning of a negative pressure the answer would be that this is pure mathematics, or it is based on other cosmological scale. The relation between the two scales is unknown. Until now we know that the vacuum has a pressure $p \approx 0$. We tried to give a physical answer to the question of the negative pressure.

a. Imagine a reference volume V of an inox strong non-elastic outer wall and lined to the inner surface of it with a table of fibrous weak material of some mm thickness of volume V_1, not withstanding great sucking up flows, that the final volume is decreased to $V_0 = V - V_1$. Thus

$$V_0 < V \tag{4.1}$$

b. Now connect the reference volume to a strong pump, who is part of our closed system and try to suck up all the molecules of the volume V_0 out. As in the end all molecules (except the fibrous) are taken out, the reference volume will become V_0 and the pressure gets $p = \varepsilon$, an infinitesimal number, (according to Nerst it is impossible to reach $p = 0$).

c. Continue the pumping and begin to suck up the fibrous molecules of the table out. As in the end all the fibrous molecules are taken out the reference volume will become the initial V and the pressure gets $p_V = \varepsilon_V$, again an infinitesimal number.

d. The whole process is completed quickly by applying great speeds of mass transfer. Because of the isothermic process we have $pV_0 =$

$p_V V$. Thus $p_V = pV_0 / V < p$. If we say that the today's matter density (S. Weinberg)

$$\rho_M = 0.92 \times 10^{-26}\ \mathrm{Kg/m^3} \qquad (4.2)$$

corresponds to pressure $p = 0$ then we can say that $p_V < p = 0$ is a negative pressure, in accordance to Eq. 2.3.

We conclude that a negative pressure is the result of the motion of some matter of the reference volume outwards, thus to an expansion.The expansions-work is

$$A = -p_V V_1 > 0.$$

In accordance with Eq. 2.3, where $V_1 \triangleq V_V$, the vacuum at the Big-Bang.

Thus we come to the result that the cosmologists define the void room as having negative pressure p_V, while the room with matter density $\rho_M = 0.92 \times 10^{-26}\ \mathrm{Kg/m^3}$ is defined as having $p = 0$. The scale of the pressure in the cosmologist language is a simple shift downwards, relatively to the common pressure p.

5. THE BIRTH OF THE VACUUM ENERGY AND THE INFLATION

The energy ΔE given by the pump (part of our closed system) to extract all the bright mass with energy density ρ_{BM} out of the reference volume and building a negative pressure, is consumed to generate a heat Q in the pump and produce an expansions energy A. The sum

$$\rho_{VBM} = \Delta E/V = (A + Q)/V + \rho_{BM} \tag{5.1}$$

is the vacuum energy density of the bright matter and V is the reference volume.

We assume that in our vicinity is no dark matter (DM), the nearest being at a distance of redshift $z = 0.296$, (Sec. 14) and is measured via the gravitational lenses, as we will describe in the Sec. 23.

The middle matter density is the sum of bright matter (BM) and dark matter (DM).

If we add to the vacuum energy density ρ_{VBM} the vacuum energy density of the dark matter $\rho_{VDM} = \rho_{VBM}\rho_{DM}/\rho_{BM}$, which is far away and did not take part in the process, we become the total vacuum energy density $\rho_{VE} = \rho_{VBM} + \rho_{VDM}$ of the reference volume. We transfer the concept of the vacuum energy density ρ_{VE} to the universe.

The expansion is a quick process causing the Inflation, while the undercooling of the universe is a slow process continuing until today (Sec. 15) .

Today the universe consists of the following energy densities (Peebles & Rata, 2003):

$\rho_{DE} \approx 68.3\%$, $\rho_{BM} \approx 4.9\%$, $\rho_{DM} \approx 26.8\%$, $\rho_R \approx 0$, (DE=Dark Energy, BM=Bright Matter, DM=Dark Matter, R=Radiation). The sum of bright and dark matter mean densities is $\rho_M \approx 0.92 \times 10^{-26}$ Kg/m^3.

6. THE EVOLUTION OF THE UNIVERSE

6.1 The Robertson-Walker metric and the geodesic

The minimum distance between two near points P_1 and P_2 on the surface of a curved worldline, connecting two events in spacetime with no mass present,we use the metric for flat spacetime

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2, (c = 1)$$

where we adopt the sign convention of the Minkowski tensor

$$\eta_{\mu\nu} = \text{diag.}(-1,1,1,1).$$

Integrating ds gives the total distance along a worldline AB

$$\Delta s = \int_A^B \sqrt{ds^2}.$$

The distance measured between two events, A and B, in a reference frame for which they occur simultaneously ($t_A = t_B$) is the proper distance

$$\Delta L = \sqrt{(\Delta S)^2}$$

In a homogenous and isotropic universe, although the curvature of space may change with time, it may have the same value everywhere at a given time since the Big-Bang.

On the surface of a sphere (Fig. 6.1), the curvature is defined as $K = 1/R^2$. But a more general expression for curvature in a 2-D space is

$$K = \frac{3}{\pi} \lim((2\pi D - C_{exact})/D^3) \text{ for } D \to 0.$$

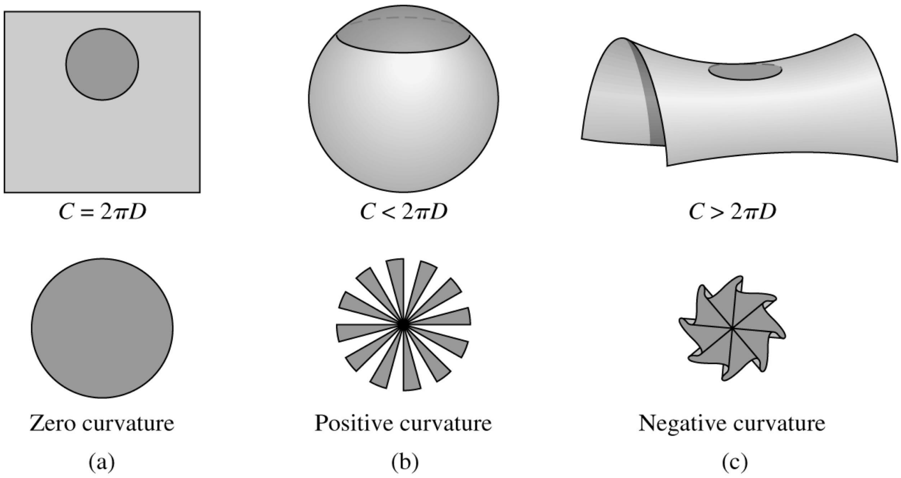

Fig. 6.1

The distance between two points, P_1 and P_2 on a sphere is given by Fig. 6.2.

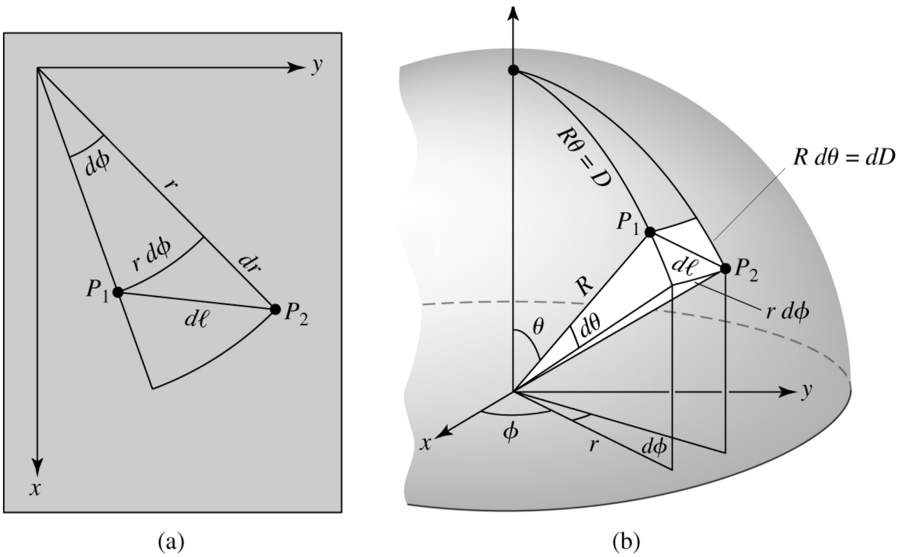

Fig.6.2

$$(dl)^2 = (Rd\theta)^2 + (rd\varphi)^2$$

where $r = R\sin\theta$, so $dr = R\cos\theta d\theta$ and

$$\mathrm{Rd}\theta = \mathrm{dr}/\cos\theta = \mathrm{Rdr}/\sqrt{\mathrm{R}^2 - \mathrm{r}^2} = \mathrm{dr}/\sqrt{1 - \mathrm{r}^2/\mathrm{R}^2}$$

so that

$$(\mathrm{dl})^2 = \mathrm{dr}^2/(1 - \mathrm{Kr}^2) + (\mathrm{rd}\varphi)^2$$

in terms of plane polar coordinates r, φ and the curvature K of two dimensional surface.

This can be extended to 3-D by changing from polar to spherical coordinates,

$$(\mathrm{dl})^2 = \mathrm{dr}^2/(1 - \mathrm{Kr}^2) + (\mathrm{rd}\theta)^2 + (\mathrm{rsin}\theta\ \mathrm{d}\varphi)^2 \qquad (6.1.1)$$

where r is now the radial coordinate (M. Pettini).

The above Eq. 6.1.1 shows the effect of the curvature of our three-dimensional universe on spatial distances.

The final step towards the spacetime metric involves the inclusion of time. In an isotropic and homogenous universe, there is no reason why time should pass at different rates at different locations; thus the temporal term should be just cdt. With $c = 1$ the metric then becomes

$$(\mathrm{ds})^2 = -(\mathrm{dt})^2 + (\mathrm{dl})^2$$

and the proper distance is just $\Delta \mathrm{L} = \sqrt{(\Delta S)^2}$ with $\mathrm{dt} = 0$.

In an expanding universe we write the radial coordinate r(t), in the rest frame of reference, as function of the comoving coordinate x

$$\mathrm{r(t)} = \mathrm{a(t)x}.$$

Because the expansion of the universe affects all of its geometric properties, including its curvature, it is also useful to define the timedependent curvature in terms of the scale factor and a time-independent constant k

$$K(t) = k/a^2(t).$$

With these substitutions for r and K, we finally arrive at the important Robertson-Walker metric

$$(ds)^2 = -(dt)^2 + a^2(t)[dx^2/(1-kx^2) + (xd\theta)^2 + (x\sin\theta\, d\varphi)^2]$$

which is more usually written in the form

$$(ds)^2 = -(dt)^2 + a^2(t)[(dr^2/(1-kr^2)) + r^2(d\theta^2 + \sin^2\theta d\varphi^2)]$$

where, through a new change of notation, r now indicates comoving radial distance.

The Robertson-Walker metric is diagonal, with

$$g_{00} = -1, g_{rr} = a^2(t)/(1-kr^2), g_{\theta\theta} = a^2(t)r^2, g_{\varphi\varphi} = a^2(t)r^2\sin^2\theta \qquad (6.1.2)$$

In cartesian coordinates

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu,$$

where the space curvature is involved in the metric tensor $g_{\mu\nu}$.

In an expanding space with expansion factor a(t), curvature k and a Robertson-Walker metric the squared distance is

$$ds^2 = dt^2 - a^2(t)[dx^2 + k(xdx)^2/(1-kx^2)] =$$

$$dt^2 - a^2(t)\widehat{g_{ij}}dx^i dx^j,$$

where the metric $g_{\mu\nu}$ in cartesian coordinates is

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 \\ & \\ 0 & \alpha^2\,(\delta_{ij} + kx^i x^j\,/\,(1\text{-}kx^2)) \end{pmatrix} = \begin{pmatrix} -1 & \\ & \\ 0 & \alpha^2\,\widehat{g_{ij}} \end{pmatrix}$$

with $g_{ij} = a^2(t)[\delta_{ij} + kx^i x^j/(1 - k\mathbf{x}^2)]$, $g_{i0} = g_{0i} = 0$, $g_{00} = -1$, $\widehat{g_{ij}} = [\delta_{ij} + kx^i x^j/(1 - k\mathbf{x}^2)]$, $\mu, \nu = 0, 1, 2, 3$, $i, j = 1, 2, 3$.

The meaning of the Robertson-Walker metric with an expansion factor a(t) can be clarified in spherical coordinates by calculating the proper distance l at time t from the origin to a comoving object at radial coordinate r in spherical coordinates .

If a particle is falling opposite to the z-direction ($\theta = \varphi = 0$)

$$ds^2 = dt^2 - a^2dr^2/(1 - kr^2) = dt^2 - dl^2, \text{ where}$$

$$dl = a(t)dr/\sqrt{1 - kr^2}$$

is the proper distance differential.

The proper distance is then

$$l(r,t) = a(t)\int_0^r dr\,/\sqrt{1 - kr^2} = a(t)\begin{cases} \sin^{-1} r \text{ for } k = 1 \\ r \text{ for } k = 0 \\ \sinh^{-1} r \text{ for } k = -1 \end{cases}$$

k stands for the space curvature.

$k = 0$ for three- dimensional euclidean (flat) space.

$k = 1$ for a spherical space and

$k = -1$ for a hyperbolical space.

In a comoving object, r is time independent, so the proper distance from us increases or decreases with a(t). The rate of change of any such proper distance l(t) is

$$\dot{l}(t) = \dot{a}(t)l(t)/a,$$

where $\dot{a}(t)$ means the time derivative of a(t).

A particle at rest in these coordinates will be described in co-moving coordinates. Because $g_{00} = -1$, the quadrat of the proper time interval for a co-moving clock is just

$$d\tau^2 = -g_{\mu\nu}(x)dx^\mu dx^\nu = dt^2 - a^2(t)[dr^2/(1-kr^2) + r^2 d\Omega],$$

where $d\Omega = d\theta^2 + \sin^2\theta d\varphi^2$.

The equation of motion of freely falling particle (or photon) in a curved space is the geodesic

$$d^2x^i/du^2 + \Gamma^i_{kl}(dx^l/du)dx^k/du \qquad (6.1.3)$$

where Γ^i_{kl} is the Christoffel affine connection and u a parameter along the space-time curve, proportional to the proper time τ, or the length of the curve and x^i a cartesian component. The geodesic means that the integral $\int dt$ is stationary under any infinitesimal variation of the path that leaves the endpoints fixed.

Following S. Weinberg the non-zero components of the Christoffel tensor are

$$\Gamma^0_{ij} = a\dot{a}[\delta_{ij} + Kx^i x^j/(1-Kx^2)] \qquad (6.1.4)$$

$$\Gamma^i_{0j} = \dot{a}\delta_{ij}/a \qquad (6.1.5)$$

$$\Gamma^i_{jl} = \widehat{\Gamma}^i_{jl} = K\widehat{g_{jl}}x^i \qquad (6.1.6)$$

We can use these components of the Christoffel tensor to find the motion of a particle that is not at rest.

The quantity with non-zero mass m_0

$$P = m_0\sqrt{g_{ij}dx^i dx^j/d\tau d\tau} \qquad (6.1.7)$$

where $d\tau$ is the proper time $d\tau = dt\sqrt{1-v^2}$ in a locally inertial comoving cartesian coordinate system, for which $g_{ij} = \delta_{ij}$ and $v^i = dx^i/dt$ is the scalar magnitude of the momentum and so invariant.

Thus we can evaluate it as well in comoving Robertson-Walker coordinates, but also in a spatial coordinate system in which the particle is near the origin, where $g_{ij} = \delta_{ij} + O(x^2)$ and we can therefore ignore the spatial components of the tensor Γ^i_{jk} ($= \widehat{\Gamma}^i_{jl} = 0$). At first let's calculate the rate of change of $(dx^i/d\tau)^2$,

$$\frac{d}{d\tau}(dx^i/d\tau)^2 = 2(dx^i/d\tau)d^2x^i/d\tau^2.$$

The geodesic (6.1.3) yields

$$d^2x^i/d\tau^2 = -(2/a)(da/dt)(dx^i/d\tau)(dt/d\tau),$$

Multiplying with $d\tau/dt$ gives

$$d/dt(dx^i/d\tau) = -(2/a)(da/dt)(dx^i/d\tau),$$

whose solution is

$$dx^i/d\tau \sim a^{-2}(t).$$

Setting this in Eq. 6.1.7 with a Robertson-Walker metric $g_{ij} = a^2(t)\delta_{ij}$ we have

$$P(t) \sim 1/a(t).$$

This holds for any non-zero mass, as much as small it may be compared to the momentum. Hence, however for photons both m_0

and $d\tau$ vanish, Eq. 6.1.7 is still valid.

Isotropy claims that the mean value of the matter current $\mathbf{J}$ of galaxies vanish and homogeneity that the mean value of any scalar n to be a function only of time, so the current $J^{\mu} = (J^0, \mathbf{J})$ of galaxies, baryons etc. has the components

$$J^0 = n(t), J^i = 0, \tag{6.1.8}$$

with n(t) the number of galaxies, baryons etc. per volume in a comoving frame of reference.
If J^{μ} is conserved, then the covariant derivative vanishes

$$0 = J^{\mu}_{;\mu} = \partial J^{\mu}/\partial x^{\mu} + \Gamma^{\mu}_{\mu\nu} J^{\nu} = dn/dt + \Gamma^{i}_{i0} n = dn/dt + 3(da/dt)n/a.$$

$$\text{So} \qquad n(t) = \text{constant}/a^3(t).$$

Similarly, isotropy requires the mean value of any three tensor t^{ij} at $\mathbf{x} = \mathbf{0}$ to be proportional to δ^{ij} and hence to g^{ij}, which equals to $a^{-2}\delta^{ij}$ at $\mathbf{x} = \mathbf{0}$.
Thus the energy-momentum tensor takes the form

$$T^{00} = \rho(t), T^{0i} = 0, T^{ij} = \delta^{ij} a^2(t) p(t) \tag{6.1.9}$$

with p(t) the pressure in the space.

The momentum conservation law claims the vanishing of the covariant derivative of the energy-momentum tensor

$$T^{i\mu}_{;\mu} = T^{i\mu}_{,\mu} + \Gamma^{i}_{\nu\mu} T^{\nu\mu} + \Gamma^{\mu}_{\nu\mu} T^{i\nu} = 0$$

near the origin ($\Gamma^{i}_{lk} = 0$) are for the Robertson-Walker metric tensor automatically satisfied, Eq. 6.1.9, but the energy conservation law yields

$$0 = T^{0\mu}_{;\mu} = \partial T^{00}/\partial t + \Gamma^0_{ij}T^{ij} + \Gamma^i_{i0}T^{00} = d\rho/dt + 3\dot{a}(p+\rho)/a,$$

the continuity equation

$$d\rho/dt + 3\dot{a}(p+\rho)/a = 0. \qquad (6.1.10)$$

Setting $w = p/\rho$ the solution of the continuity equation gives

$$\rho(t) = \rho_0 a(t)^{-3(1+w)} \qquad (6.1.11)$$

where $w = p/\rho$ is time-independent, corresponding to different epochs.
$w = -1$, corresponds to the vacuum epoch ($p = -\rho_V$). At this epoch the scale factor is

$$a(t) = \exp(Ht). \qquad (6.1.12)$$

$w = 1/3$ corresponds to the radiation epoch ($p = \rho_R/3$) and $w = 0$ to the matter epoch ($p = 0$), $\rho = \rho_M$.

6.2 The Einstein equation

The universe, containing matter and pressure gradients, having a metric $g_{\mu\nu}$ evolves according to Einstein equation

$$G_{\mu\nu} = -8\pi G T_{\mu\nu} \qquad (6.2.1)$$

where

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R,$$

$R_{\mu\nu}$ is the Ricci tensor and the scalar R respectively.

$$R_{\mu\nu} = \partial\Gamma^\lambda_{\lambda\mu}/\partial x^\nu - \partial\Gamma^\lambda_{\mu\nu}/\partial x^\lambda + \Gamma^\lambda_{\mu\sigma}\Gamma^\sigma_{\nu\lambda} - \Gamma^\lambda_{\mu\nu}\Gamma^\sigma_{\lambda\sigma}$$

$$\Gamma^\mu_{\nu\kappa} = \frac{1}{2}g^{\mu\lambda}[\partial g_{\lambda\nu}/\partial x^\kappa + \partial g_{\lambda\kappa}/\partial x^\nu - \partial g_{\nu\kappa}/\partial x^\lambda]$$

is the Christoffel affine connection and $T_{\mu\nu}$ is the energy-momentum tensor.

The nonvanishing affine connections are

$\Gamma^0_{11} = a\dot{a}/(1-kr^2), \Gamma^0_{22} = a\dot{a}r^2, \Gamma^0_{33} = a\dot{a}r^2\sin^2\theta$

$\Gamma^1_{01} = \Gamma^1_{10} = \Gamma^2_{02} = \Gamma^3_{03} = \dot{a}/a$

$\Gamma^1_{22} = -r(1-kr^2), \Gamma^1_{33} = -r(1-kr^2)\sin^2\theta$

$\Gamma^2_{12} = \Gamma^2_{21} = \Gamma^3_{13} = \Gamma^3_{31} = 1/r$

$\Gamma^2_{33} = -\sin\theta\cos\theta, \Gamma^3_{23} = \Gamma^3_{32} = \mathrm{ctg}\theta$

The Ricci tensors being symmetric can be diagonalised, so

$R_{00} = -3\ddot{a}/a$

$R_{11} = [(a\ddot{a} + 2\dot{a}^2 + 2k)/(1-kr^2)]$

$R_{22} = r^2[a\ddot{a} + 2\dot{a}^2 + 2k]$

$R_{33} = r^2[a\ddot{a} + 2\dot{a}^2 + 2k]\sin^2\theta.$

The components of the Robertson-Walker metric are given by Eq. (6.1.2).

For a comoving observer the tensor components T_{00} and T_{11} of the Eq.6.2.1 are

$$T_{00} = \rho \text{ and } T_{11} = pa^2/(1-kr^2) \tag{6.2.2}$$

where ρ and p are the mass density and the pressure respectively.

The corresponding tensor components of the Einstein equation after rather lengthy derivation are

$$G_{00} = 3a^{-2}(\dot{a}^2 + k), \tag{6.2.3}$$

And $G_{11} = -[2a(d^2a/dt^2) + \dot{a}^2 + k]/(1-kr^2)$

With substitution of (6.2.2) and (6.2.3) in (6.2.1) we become the Friedmann Eq.

$$\dot{a}^2/a^2 + k/a^2 = 8\pi G\rho/3 \qquad (6.2.4)$$

and the equation

$$2\ddot{a}/a + \dot{a}^2/a^2 + k/a^2 = -8\pi Gp \qquad (6.2.5)$$

Substracting the Friedmann Eq. from the Eq. 6.2.5 we get

$$\ddot{a}/a = -4\pi G(\rho + 3p)/3 \qquad (6.2.6)$$

which shows that the acceleration of the expansion decreases with increasing pressure and energy density.

Einstein expressed the space containing matter density ρ, which interact with attraction forces between them and pressure gradients, which creates matter currents, in the $T^{\mu\nu}$ tensor. So because usually $p \ll \rho$, to counterbalance the attraction a positive pressure p_V in form of a cosmological energy density ρ_Λ or vacuum energy density ρ_V, as tensor $\Lambda g^{\mu\nu}$ was added in the Einstein equations next to the energy momentum tensor $T^{\mu\nu}$. When the matter included in the energy momentum tensor disappeared at the Big Bang, the cosmological constant in the Einstein Eq. remained.

If p_V>0, this will explain the expansion of the universe.

Setting $\Lambda g^{\mu\nu}$ in the opposite side of $T_{\mu\nu}$ on the Einstein equation,as it is usually done, the positive pressure will become (Eq.2.3) negative.

If we express the total energy momentum tensor $T_{\mu\nu}$ as the sum of a possible vacuum term $p_V g_{\mu\nu} = -\rho_V g_{\mu\nu}$ and a term $T^M_{\mu\nu}$ arising from matter (including radiation) then the Einstein equations take the form

$$R_{\mu\nu} - g_{\mu\nu}R_{\lambda}^{\lambda}/2 = -8\pi G(T_{\mu\nu}^{M} - \rho_V g_{\mu\nu}).$$

Thus the Einstein equations can be written as

$$R_{\mu\nu} - g_{\mu\nu}R/2 - \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}^{M} \qquad (6.2.7)$$

$$\text{where} \qquad \Lambda = 8\pi G\rho_V.$$

The quantity Λ is known as the cosmological constant.

Cosmological observations indicate that Λ is at least of the order of $1.1 \times 10^{-52}\ \mathrm{m}^{-2}$ in geometrical units ($c = G = 1$), $1.1 \times 10^{-8}\ \mathrm{Jm}^{-3}$ in SI units and $6.3 \times 10^{-44}\ \mathrm{GeV}^4$in natural units ($c = \hbar = k_B = 1$) (see Sec. 25).

In physical local problems with small time duration we can consider the universe static, $a = 1$, and in physical problems in one small part of the universe, the cosmological constant usually takes the value $\Lambda = 0$, (N.K. Spyrou).

The discovery since 1998 that the universe expands accelerating has brought the cosmology into a new era, called "dark energy" with energy density ρ_Λ.

The dark energy density $\rho_\Lambda$was found to be equal to the vacuum energy density ρ_V.

Considering the Einstein equation in the form (6.2.7), Eq. 6.2.4 and 6.2.6 become

$$\dot{a}^2 = -k + (8\pi G\rho - \Lambda)a^2/3 \qquad (6.2.8)$$

$$\ddot{a} = -[4\pi G(\rho + 3p) + \Lambda]a \qquad (6.2.9)$$

The Friedmann equation with the Hubble parameter $H = \dot{a}/a$, and the cosmological constant, defines how the energy density $\rho(t)$ drives the universes evolution .

6.3. The different energy forms ratios and the estimation of k

During the Big-Bang a collision of stars took place and a great amount of heat was released, transforming the solid masses into plasma that flied far away leaving a vacuum at collision's center.

The separation to different epochs is a mathematical consequence of defining w time- independed. In reality all epochs give a distribution to the following epochs .

We define a critical matter density, with $H = \dot{a}/a$ the Hubble constant

$$\rho_C = 3H^2/8\pi G,$$

the today's universe mean matter density and the mean density parameters

$$\Omega_0 = \rho_0/\rho_C, \text{ where } \rho_0 = \rho_M + \rho_R, \Omega_M = \rho_M/\rho_C, \Omega_R = \rho_R/\rho_C,$$

$$\Omega_\Lambda = \rho_\Lambda/\rho_C, \text{ and } \Omega_K = -k/a^2H^2.$$

$$\Omega_M + \Omega_R + \Omega_\Lambda + \Omega_K = 1.$$

M, R, Λ, K stands for matter, radiation, dark energy and the curvature, respectively.

It is $\rho_M + \rho_R + \rho_\Lambda + \rho_K = \rho_C$.

From (6.1.11) follows that

$$\rho_M = \rho_{M0} a^{-3}, \rho_R = \rho_{R0} a^{-4}, \rho_\Lambda = \rho_{\Lambda 0} \rho^{-3(1+w)}, \rho_K = \rho_{K0} a^{-2} \text{ and}$$
$$\rho(t) = \rho_{M0} a^{-3} + \rho_{R0} a^{-4} + \rho_{\Lambda 0} a^{-3(1+w)} + \rho_{K0} a^{-2}.$$

This parametrisation has the advantage of letting only one scale in the models and the Hubble constant H. All other quantities can be given as dimensionless ratios. The term ρ_C is the critical today's matter density $\rho_C = 3H_0^2(8\pi G)^{-1} = 0.92 \times 10^{-26}$ Kg/m³.

Today the pressure is too small: $p/\rho = 10^{-4}$. Let us take $p = 0$.
At the present epoch $t = t_0$ $(p = 0)$ (6.2.9) and (6.2.8) are given by

$$\ddot{a} = -(4\pi G \rho_0\, a_0 + \Lambda a_0)/3, \qquad (6.3.1)$$

$$\dot{a}^2 = 8\pi\, G \rho_0 a_0^2 - k - \Lambda a_0^2/3 \qquad (6.3.2)$$

They can be rewritten in terms of the Hubble constant H_0 and the "deceleration" parameter $q_0 = -\ddot{a}a/\dot{a}^2$, at $t = t_0$ the today's time.

$$4\pi G \rho_0/3 = q_0\, H_0^2 + \Lambda/3,$$

$$k/a_0^2 = H_0^2(2q_0 - 1) - \Lambda.$$

Eliminating Λ we become

$$k/a_0^2 = H_0^2(3\Omega_0 - 2q_0 - 2)/2$$

and setting $\Lambda = 3H_0^2\Omega_\Lambda$, from the equations (6.3.1) and (6.3.2) we have $\Omega_\Lambda = \Omega_0/2 - q_0$.

Then $k = (a_0^2 H_0^2)(\Omega_0 + \Omega_\Lambda - 1)$.

$$\text{If: } a_0^2H_0^2(\Omega_0+\Omega_{\Lambda}\text{-}-1)\begin{cases}\geq 1, \text{then } k\geq 1 \text{ and calibrated } k=1\\ =0, \text{then } k=0\\ <1, \text{then } k<1 \text{ and calibrated } k=-1\end{cases}$$

Since $\Omega_\Lambda = -q_0 + \Omega_0/2$ and $\Omega_0 = \rho_0/\rho_C$, where $\rho_0 = \rho_M + \rho_R$, are measurable quantities, it is possible to determine whether our cosmos is a spherical, flat or hyperbolic one.

We can of course define these quantities for all cosmic epochs, which we will do later (Chapter 18) in order to estimate numerically the age of our universe. At the same Chapter we will see that the measurements give $\Omega_0 = 0.24$, $\Omega_\Lambda = 0.76$, $k = 0$, indicating for the today's $k = 0$.

We can rewrite the Hubble parameter as

$$H(a)^2 = H_0^2\left(\Omega_K a^{-2} + \Omega_M a^{-3} + \Omega_R a^{-4} + \Omega_\Lambda a^{-3(1+w)}\right), \qquad (6.3.3)$$

where the four currently hypothesised contributors to the energy density of the universe are curvature, matter, radiation and dark energy. Each of the components decreases with the expansion of the universe (the scale factor increases), except perhaps the dark energy term. It is the values of these cosmological parameters which physicists use to determine the acceleration of the universe.

The acceleration equation describes the evolution of the scale factor with time

$$\ddot{a}/a = -4\pi G(\rho + 3p)/3.$$

According to the theory of cosmic inflation the very early universe underwent a period of very rapid, quasi- exponential expansion, while the time-scale for this period of expansion was far shorter than that of the dark energy-dominated expansion.

The accelerating expansion means that the second time derivative of the cosmic scale factor $\ddot{a}$ is positive which is equivalent to the deceleration parameter being negative.

However, note that this does not imply that the Hubble parameter is increasing with time. Since the Hubble parameter is defined as $H(t) = \dot{a}(t)/a(t)$, it follows that the derivative of the Hubble parameter is given by

$$dH/dt = -H^2(1 + q_0)$$

So the Hubble parameter is decreasing with time (very slowly) unless $q_0 < -1$. Observations prefer $q_0 \approx -0.55$, which implies that dH/dt is negative.

Essentially this implies that the cosmic recession velocity of any particular galaxy is increasing with time, but its velocity/ distance ratio (H) is still decreasing.

Thus different galaxies expanding across a radius of a sphere cross the surface more slowly at earlier times.

It is seen from the above that the case of “zero acceleration” corresponds to $a(t)$ is a linear function of t, $q_0 = 0$ and $H(t) \sim 1/t$.

For a spherical ($k = 1$) and static ($a = a_E = \text{const.}$ and $\Lambda = 0$) universe from the Eq.6.1.10 and 6.1.8 means that for a stable universe must

$$3p + \rho = 0 \text{ and } k = 8\pi\rho G a^2/3.$$

If $\rho = \rho_M + \rho_V$, ($\rho_V = 0$), $p = -\rho_V$ it follows that $\rho_M = 2\rho_V$ and $\rho = 3\rho_V$.

$$\text{So} \quad a_E = 1/\sqrt{8\pi G\rho_V} = 1/\sqrt{\Lambda}$$

a very popular solution at the beginning (Einstein model).

This was an unstable universe because for an infinitesimal positive disturbance $\delta\rho$ added to ρ_V a_E decreases. For a negative disturbance added to ρ_Λ a_E increases, so a_E does not remain constant (Fig. 6.3).

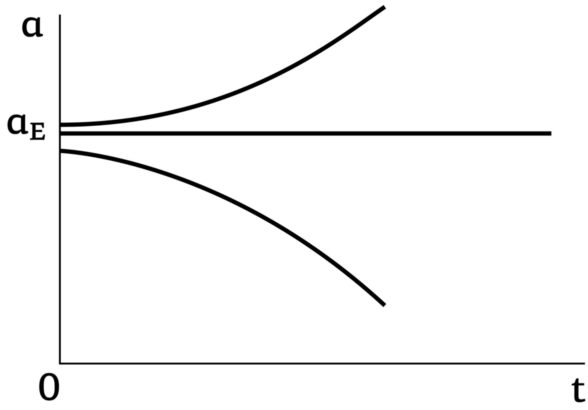

Fig.6.3

To make the universe stable a cosmological constant Λ was necessary in the Einstein equations, something Einstein at the beginning did, but later he removed it, admitting that this was his greatest mistake. Finally he added it.

Starting at $a = 0$ and approaching a_E, or starting at $a = a_E$ and go to infinity, we have the Eddington-Lemaitre model.

Starting at $a = 0$ and diverging to Infinity as $t \to$ infinity we have the Lemaitre model and finally starting at $a = a_E$, spending a long time at a_E and then expanding to infinity, we have the de Sitter model with $k = 0$ and $a \sim e^{Ht}$ (Fig 6.4).

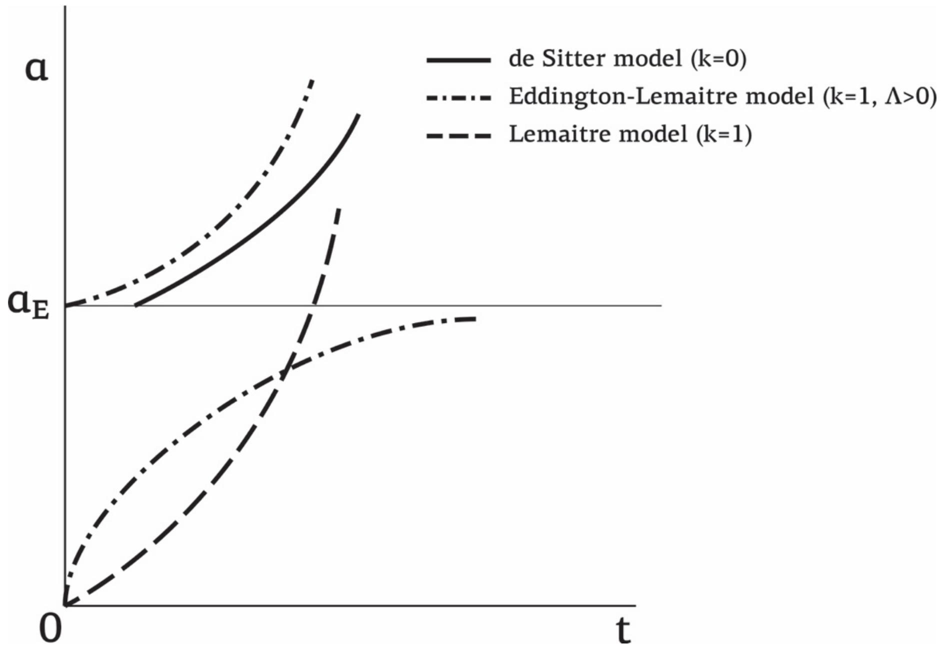
a
a_E
0
t
de Sitter model (k=0)
Eddington-Lemaitre model (k=1, Λ>0)
Lemaitre model (k=1)

Fig.6.4

7. THE EARLY VACUUM PERIOD

This period is estimated in the time before T_H, (H stands for Higgs) at temperature $T \approx 10^{12}$ K (Weinberg), where the neutron was changed to proton and electron. The Lorentz invariance of the energy momentum tensor in a locally inertial comoving coordinate system in the vacuum dominated epoch $(p = -\rho)$ requires that $T^{\mu\nu}$ in a homogeneous isotropic space to be proportional to the Minkowski metric

$$\eta^{\mu\nu} = \mathrm{diag}(-1,1,1,1),$$

of the form

$$T^{\mu\nu} = \mathrm{diag}(\rho, -p, -p, -p)$$

In that epoch obviously was $\rho > 0$. So because of $p + \rho = 0$, $p < 0$.

During this period the universe was pervaded by a Higgs field φ and an electromagnetic field A^{μ}. The density of this radiation field as function of the temperature was

$$\rho_R = T^4, \tag{7.1}$$

The potential $V(\varphi)$ of it sitting at $t = 0$ on a minimum of a mexican hat, Fig. 9.1,

$$d\varphi/dt = 0, \rho(\varphi) = (½)d\varphi/dt + V(\varphi).$$

Therefore

$$\rho_R(t = 0) = V(0) \sim T^4 \approx 10^{48}\mathrm{GeV}^4. \tag{7.2}$$

During the inflation period the coefficient of the universe expansion increased as $a(t) \sim \exp(Ht)$, while H was constant. This inflation period came before the radiation and today's matter epoch. The value of the dark energy measured to be today $\approx$68.3% of the total energy. The bright mass density is 4.9% while the dark mass density is 26.8% and the radiation energy $\approx$0. During the expansion the absolute value of the negative pressure became greater and today is $\approx$0.

Through the Higgs mechanism matter stars are built and the free Volume of the universe decreases. So the pressure increases. The galaxies following the Herzsprung-Russell diagram (Sec. 20) got at a later time white dwarfs and black holes, increasing thus the volume of the universe, parallel to the increasing because of the expansion.

Until the building of atoms after 3.8×10^5 years the universe was totally dark.

8. A MODEL FOR THE BIG-BANG

We assume that two bodies B_1 and B_2 have been found close to each other, so that the cosmological constant, according to Sec. 2, can be set $\Lambda = 0$ and they come to a collision.

8.1 Plastic collision between two cold bodies

After the Big-Bang the temperature was $T \approx 10^{12}$, (S. Weinberg) and the radiation density was according to Eq.7.1 $\rho_R = T^4 \approx 10^{48}$ GeV^4, in natural units ($c = \hbar = \varepsilon_0 = k_B = 1$, c the light velocity, $\hbar = h/2\pi$, h the Planck's, ε_0 the electric and k_B the Boltzmann constant).

Let us assume a plastic collision between two bodies with a mass density similar to that of the earth with $\rho_{MG} = 5.4 \times 10^3$ Kg/m^3 and the energy density $\rho_{EG} = 4.6 \times 10^{-17}$ GeV^4. The density of the two bodies after the collision would be (Eq 2.2)

$$\rho_{EG} \approx 10^{-17+x/2}\ \text{GeV}^4, \text{(Sec. 24)}.$$

If we set $\rho_R = T^4 \approx 10^{48}\text{GeV}^4 \approx 10^{-17+x/2}$ we take $x = 130$ and the velocities of the two bodies before the collision would have a value of $v = 1 - 10^{-130}$. These values are far too great and must be rejected because they don't fulfil the condition from Eq.2.1.

8.2 Plastic collision between two hot bodies

1. Plastic collision between two pulsars.

Two pulsars (neutron stars) with temperature $T = 10^6$ K and energy $\rho_{RG} = 10^6$GeV will produce an energy density after the collision of (Eq. 2.2) $\rho_{EG} = 10^{24+x/2}\text{GeV}^4, (c = 1)$. If we equalise the energy density at the Big-Bang with the heat energy density of the two pulsars (neglecting their spin and rotation of each around the other) at the collision we get $\rho_R = T^4 = 10^{48}GeV^4 = 10^{24+x/2}GeV^4$ and the solution is $x = 48$ and the velocities of the two bodies before the collision would be $v = 1 - 10^{-48}$. These values must be rejected because of the condition (2.1).

2. Plastic collision between two red Giants.

Let us assume that the two bodies were red Giants, their states were plasma with temperature $T = 2 \times 10^8 K$ and energy $\rho_{RG} = T\, GeV$, (Atlante del cielo 2008), then the energy density of the two bodies after the collision would be (Eq 2.2) $\rho_{EG} = 3.2 \times 10^{33+x/2}GeV^4$, ($c = 1$, ρ_{EG} = the energy density of the red Giants). If we equalise the energy density at the Big-Bang $\rho_R = T^4 = 10^{48}GeV^4$ with the energy density of the red Giants after the collision we get $\rho_R = T^4 = 10^{48}GeV^4 = 3.2 \times 10^{33+x/2}$ and the solution is $x = 30$ and the velocities of the two bodies before the collision would be $v = 1 - 10^{-30}$. These values must be rejected because of the condition (2.1).

If we assume that at the collision spacetime, because of fluctuation, the temperature exceeds the mean value by three orders of magnitude higher $T = 2 \times 10^{11}K$, then we come to a value of $x = 6$. The velocity of the two bodies before the collision would be $v = 1 - 10^{-6}$. This value seems reasonable, according to the condition (2.1). What is left behind is a Higg's field φ at the center of collision with a totally symmetric potential $V(\varphi = 0)$ and an electromagnetic potential A^μ (s. Sect. 10).

9. GAUGE THEORIES

The electromagnetic field can be expressed by the potential $A^\mu = (\varphi, \mathbf{A})$, where φ is a Coulomb potential and $\mathbf{A}$ a vector potential. It obeys the Maxwell equation in vacuum

$$\partial_\mu F^{\mu\nu} = 0,$$

$$\text{where } F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu.$$

The Maxwell equation has gauge symmetry, which means that the electromagnetic field does not change by a gauge transformation

$$A^\mu(x) \to A^\mu(x) + \partial^\mu \alpha(x).$$

The gauge has to be fixed for calculating observable quantities. The usual fixing is to adopt the Lorentz gauge

$$\partial_\mu A^\mu = 0, \tag{9.1}$$

which means $\partial_\mu \, \partial^\mu \alpha(x) = 0$.

An example is the roulette at the casinos. During the rotation of the roulette every number is probable but none is definite. After the rest one number is definite by the pointer (symmetry breaking, see Sect. 10.1) and the positions of the others are also definite.

Modern physics has generalized the previous principle of symmetry in vacuum and specially to the Higgs field φ. The symmetry must hold at the beginning of the the Big-Bang at time $t = 0$. At this time the fields ψ must be locally invariant to rotations $\psi \to U(1)\psi = e^{i\theta}\psi$ and generally also to transformations $\psi \to SU(5)\psi$ of the symmetry group SU(5), which holds at times $t \approx 0$. The letter U denotes that U is a unitary matrix. A unitary matrix is one whose inverse is equal to its transpose conjugate ($U^{-1} = U^{T*}$). The letter S stands for special, which means that the determinant $(U) = 1$. Finally the number 5 defines the order 5×5 of the matrix. A Hermitian matrix H is equal to his conjugate's transpose. Now just as any complex number of modulus 1 can be written in the form $e^{i\theta}$, with a real θ, so any unitary matrix can be written in the form

$$U = e^{iH}$$

where H is Hermitian ($H^{+} = H$). Moreover, the most general U 2×2 matrix can be expressed in terms of four real numbers a_1, a_2, a_3 and θ.

$$H = \theta 1 + \boldsymbol{\tau}\mathbf{a}$$

where 1 is the 2×2 unit matrix, $\boldsymbol{\tau} = (\tau_1, \tau_2, \tau_3)$ are the Pauli matrices, $\mathbf{a}$ is the column (a_1, a_2, a_3) and $\boldsymbol{\tau}\mathbf{a}$ their dot product. Thus any unitary U 2×2 matrix can be expressed as a product

$$U = e^{i\theta} e^{i\tau a} = e^{iH}.$$

The first factor $e^{i\theta}$ is the phase transformation $U(1)$ and the second having determinant 1 is an $SU(2)$ transformation. Because the Pauli matrices are non commutative, the transformations $SU(2)$ are also not commutative (non abelian). The $SU(5)$ group involves the rotations $U(1) = e^{i\varphi}$, the $SU(2)$ and the $SU(3)$ groups (see Sect.9.2)

$$SU(5) = U(1) \otimes SU(2) \otimes SU(3).$$

9.1 Yang-Mills non abelian theory for the SU(2) group.

In the weak interaction we have the pairs (p, n) and (e, ν_e) or (μ, ν_μ) or (τ, ν_τ). Any time the combination of the (p, n) pair with one of the three generations (flavors) of (electron, neutrino)-pair*. Now suppose that we have two spin 1/2-fields ψ_1 and ψ_2 four component Dirac spinors. The Lagrangian, in the absense of any interactions, is the sum of their own Lagrangians

$$L = \sum_{i=1}^{2} [i\overline{\psi_i}\gamma^\mu \partial_\mu \psi_i - m_i \overline{\psi_i}\psi_i], \tag{9.2}$$

But we can write the Eq.9.2 more compactly by combining ψ_1 and ψ_2 into a two component column vector

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}.$$

The adjoint spinor is $\overline{\psi} = (\overline{\psi_1}, \overline{\psi_2})$

And the Lagrangian becomes $L = i\overline{\psi}\gamma^\mu \partial_\mu \psi - \overline{\psi}M\psi$ where

$$M = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$$

M is the cross-matrix. In particular, if the two masses happen to be equal, m, Eq. 9.2 reduces to

$$L = i\overline{\psi}\gamma^\mu \partial_\mu \psi - \overline{\psi}m\psi, \tag{9.3}$$

This looks just like the one particle Dirac Lagrangian. However, ψ is now a two-element column vector and L admits a more general global invariance

$$\psi \to U(2)\, \psi$$

where U is any 2×2 unitary matrix $U^+U = 1$.

* The masses of e, μ and τ are 0.511 MeV, 105.7 MeV and 1777 MeV respectively. The τ-electrons were built obviously at higher temperatures of the universe.

In the case where the two Dirac fields are the proton and the neutron the masses m_1 and m_2 are nearly equal, so the Eq.9.3 is justified, but if the two Dirac fields are the electron and the neutrino the Eq.9.3 with $m_1 = m_e$ and $m_2 = m_\nu \approx 0$ guides to a catastrophe.

That was the reason which obliged Higgs, Englert, Brout, Guralnic-Hagen and Kibble to insist the symmetry at t=0, demanding an equality of the electron and neutrino masses being zero. This has been generalized to the quarks (constituents of the proton, neutron and mesons) and the bosons $W^{\pm}$, Z to have a mass, at t=0, equal to zero, as is the mass of photon.

9.2. Chromodynamics in the strong interaction

To conclude this Chapter we mention that the SU(3) matrix represents the strong interaction and can be expressed as

$$\mathrm{SU}(3) = e^{iH}$$

$$\text{where } H = \exp(\mathbf{a}\boldsymbol{\lambda}).$$

a is an eight component vector and **λ** eight 3×3 non commutative matrices (Gell-Mann matrices). **aλ** is their dot product.

Because **λ** matrices are not commutative the transformation SU(3) acting on three quarks

$$\begin{pmatrix} \psi_{f1}^{R} \\ \psi_{f2}^{B} \\ \psi_{f3}^{G} \end{pmatrix} \rightarrow \mathrm{SU}(3) \begin{pmatrix} \psi_{f1}^{R} \\ \psi_{f2}^{B} \\ \psi_{f3}^{G} \end{pmatrix}$$

is also non commutative (non abelian).The quarks have three different colors, red, blue and green (R,B,G) and belong to one of the three generations (flavors $f = 1,2,3$) .Their symmetry had not been in doubt. The binding force between the quarks are eight massless gluons.

10. THE HIGGS-MECHANISM

10.1 The spontaneous symmetry breaking of a real field

In the Standard Model the Lagrangian for a real field φ is symmetric at $\pm\varphi$

$$L = 1/2[(\partial_\mu\varphi\,\partial^\mu\varphi - V(\varphi)].$$

The potential $V(\varphi) = -\frac{1}{2}\varphi^2\mu^2 + \lambda^2\varphi^4/4 + V(0)$ of the Higgs field φ (Higgs-Boson) with mass $m = \mu^2$ has an unstable minimum at $\varphi = 0$. The first term is the mass and the second term is the interaction. The third term is the value of the potential $V(0)$. This is equal to the cosmological constant Λ at the time t=0. Usually we start from a minimum of the potential energy, i.e. from a zero derivative of the potential, so the smallest perturbation at φ=0 breaks the symmetry leading the potential to a stable minimum $f\cdot i$ at $\varphi = +\mu/\lambda$.This symmetry breaking, analogous to the Lorentz gauge Eq.9.1, makes the calculation of the observable quantities possible.

10.2 Spontaneous symmetry breaking of a complex field, giving to the physical observables a definite value

The Lagrangian L of a complex field is the difference between the kinetic term and the potential $V(\varphi) = -\frac{1}{2}\varphi^+\varphi\mu^2 + \lambda^2(\varphi^+\varphi)^2/4 + V(0)$.

$$L = 1/2[(\partial_\mu\varphi^+\,\partial^\mu\varphi\ - V(\varphi)].$$

Usually we start from a minimum of the potential energy, i.e from a zero derivative of the potential,

$$d^2\varphi/dt^2 = \ \varphi(\mu^2 - \lambda^2\varphi^2) = 0$$

This equation has the solutions $\varphi = 0$ and $\varphi = \pm\mu/\lambda$. We interpret the solution of the Higgs Boson $\varphi = 0$ as sitting on an unstable minimum at the top of a mexican hat with angle symmetry and the solution (without restriction of the generality) $\varphi = \mu/\lambda$ on the real axis as a stable minimum, after a spontaneous symmetry breaking, at the bottom of the mexican hat, Fig 10.1.

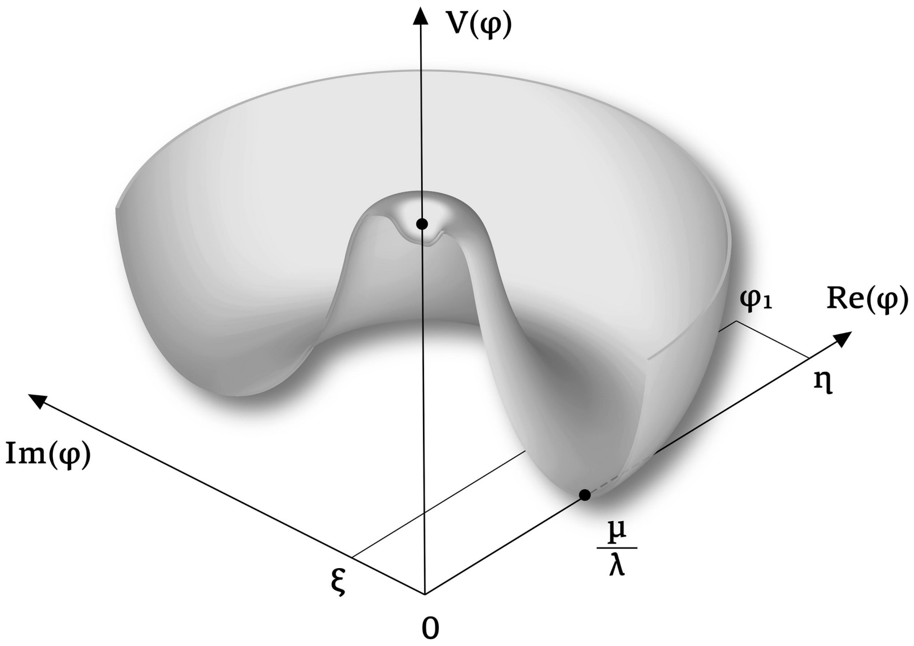

Fig. 10.1

Fig.10.1 is a double Figure. At $t = 0$ the Higgs Boson is situated at the top and at $t = \Delta t$ lies at the bottom of the hat at $\varphi = \mu/\lambda$.

Then every physical observable is definite (see Sect. 9.1).

10.3 Local gauge invariance

Local gauge invariance is needed, if scalar fields come together with electromagnetic fields, with potential A_μ. The Lagrangian has then the form

$$L = \frac{1}{2}(\partial_\mu \varphi^+ \partial^\mu \varphi + \frac{1}{2}\mu^2 \varphi^+ \varphi - \lambda^2(\varphi^+\varphi)^2/4 - F^{\mu\nu}F_{\mu\nu}/16\pi,$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$.

This invariance means that you should be able to make local gauge transformations to the fields involved in the theory without changing the physical laws. A such gauge transformation is the rotation of the field $\varphi(x) \to e^{i\theta}\varphi(x)$. If the parameter θ depends on x then we call it local, otherwise we call it global.

In quantum field theory there are two ways that scalar complex fields can show up: either directly, or as derivatives. They always show up in pairs or larger groupings, usually of a conjugate or a normal field, like $\varphi^+\varphi$. (φ^+ is the hermitian conjugate, which is the complex conjugate and transpose of a four component column field). Terms like that which involve only the fields themselves are always gauge invariant, because when you make the gauge transformation, you get the phase factor cancelled out. But if you have a term that involves derivatives, like $\partial\varphi^+ \partial\varphi$, you get additionally the derivative of the phase factor $\alpha(x)$. In order to keep the physical laws invariant we need to add to the derivatives the coupling of the electromagnetic potential A_μ to the field φ, iqA_μ. Additionally we must make the transformation $A_\mu(x) \to A_\mu(x) + q^{-1}\partial\alpha(x)$. Thus you end up to the covariant derivative $D_\mu = \partial_\mu + iqA_\mu(x)$, where the phase factor $\alpha(x)$ cancels out of $D\varphi^+ D\varphi$.

10.4 The mass-acquiring mechanism

The Lagrangian of a scalar complex field φ together with an electromagnetic field with potential $A_\mu(A = A_\mu)$ is $L = \frac{1}{2}(\partial - iqA)\varphi^+(\partial + iqA)\varphi + \frac{1}{2}\mu^2\varphi^+\varphi - \lambda^2(\varphi^+\varphi)^2/4 - F^{\mu\nu}F_{\mu\nu}/16\pi$.

The exact solutions in quantum field theory are possible only in special forms of the potential (f,i $V(r)\sim r^{-1}$) and mostly impossible. That's why, going from a known solution, f.i, $\varphi = \mu/\lambda$ we use the perturbation theory and try to continue it at a neighboring φ_1. Defining the perturbations $\eta = \varphi_1 - \mu/\lambda$ and $\xi = \varphi_2$ the Goldstone boson, the Lagrangian becomes

$$L = \left[\frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \mu^2\eta^2\right] + \left[-\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\left(\frac{q\mu}{\lambda}\right)^2 A_\mu A^\mu\right] +$$
$$\{\frac{\mu}{\lambda}q^2\eta(A_\mu A^\mu) + \frac{1}{2}q^2\eta^2(A_\mu A^\mu) - \lambda\mu\eta^3 - \frac{1}{4}\lambda^2\eta^4\} + \mu^4/4\lambda^2.$$

Now we must explain the meaning of some terms in the Lagrangian.

There are :

1. Kinetic terms involving two derivatives of the fields f.i. $\partial_\mu\eta^+\,\partial^\mu\eta$.
2. Mass terms involving products of the form $m_\eta\eta^+\eta$, or $m_A A_\mu A^\mu$.
3. Interaction terms involving products of three or more fields of the form $\sim A^2\eta, \eta^3$, or η^4.

By an astute choice of gauge, we have eliminated the Goldstone boson ξ and some offending terms in L: we are left with a single massive scalar η the Higgs boson with mass $m = \mu^2$ and a massive gauge field A^μ with mass $m = (q\mu/\lambda)^2$. One can say that the potential

A^μ has eaten the Goldstone boson. According to combination of the values of μ and λ, have the different fermions and bosons, aquired the right mass.

Just because of the change of the coordinates, a massless electromagnetic potential A^μ has acquired mass. That is in a nutshell the result of the Higgs-mechanism.

The Higgs boson discovery was announced by the ATLAS and CMS collaborations on July 2012 in Large Hadron Collider (LHC) at Geneva.

Evidence for a new particle with the mass of about 125 GeV and the properties of the Higgs boson was present in experiments, where

a. The Higgs boson is produced out of two gluons via a quantum loop process involving two top quarks.This production process has the largest cross section at the LHC (Large Hadron Collider), Fig. 10.2.

b. The second most important process is the radiation of W or Z bosons from incoming quarks, which fuse to produce a Higgs boson.

c,d. There exist two additional significant contributions of Higgs boson production with a vector boson W or Z and with a top-antitop $(t-\bar{t})$ pair.

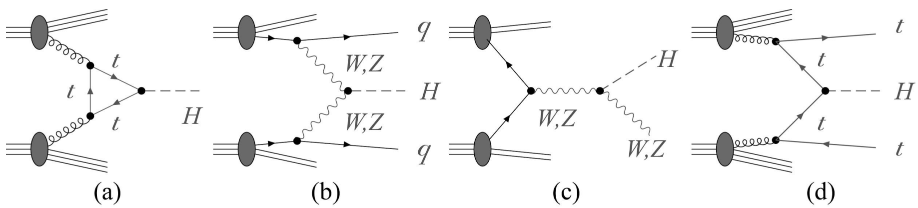

Fig.10.2

The H-boson decays naturally in different ways,mostly in $b, \bar{b}$ quarks with mean life time of10^{-22} sec.

11. THEORY OF SMALL FLUCTUATIONS

Until now we described the universe as has been isotropic and homogenous with a Robertson-Walker metric.

For $k = 0$ we write the perturbed metric as

$$g_{\mu\nu} = \overline{g_{\mu\nu}} + h_{\mu\nu},$$

the perturbed energy momentum tensor as

$$T_{\mu\nu} = \overline{T_{\mu\nu}} + \ t_{\mu\nu}$$

And the Ricci tensor as

$$R_{\mu\nu} + \overline{R_{\mu\nu}} + r_{\mu\nu},$$

where $\overline{g_{\mu\nu}}$ is the unperturbed Robertson- Walker metric

$$\overline{g_{00}} = -1, \overline{g_{0j}} = \overline{g_{j0}} = 0, \overline{g_{ij}} = a^2(t)\delta_{ij}, i.j \ = 1,2,3$$

and $h_{\mu\nu} = h_{\nu\mu}, t_{\mu\nu} = t_{\nu\mu}$ and $r_{\mu\nu} = r_{\nu\mu}$ a small perturbation.

(Here and from now on, a bar over a quantity denotes its unperturbed value.)

With these decompositions and using the Eq.6.1.10 and 6.1.8 ($k = \Lambda = 0$) the Einstein and the equations of of conservation the energy and momentum laws give the perturbation equations.We separate them in scalar $\delta\rho, \delta p, du$..., vector $\boldsymbol{\delta\nu}, \mathbf{G}, \mathbf{C}$... and tensor $\pi^T_{\mu\nu}$, $D^T_{\mu\nu}$,... (inertial momentum, gravitation radiation tensor) perturbations.

However the results are very complicated.

We work at the comoving coordinate system. Only the initial conditions depend on the direction $\mathbf{q}$ but not the Fourier components

$\delta\rho_q$, δp_q, δu_q... They depend only on q.

The advantage of the Fourier composition for instance $\delta\rho(x,t)$ (n is the number of independent solutions, $\alpha_n(\mathbf{q})$ a normalized coefficient of each solution, depending on the initial conditions)

$$\delta\rho(\mathbf{x},t) = \sum_n \int d^3q \exp(i\mathbf{q}\mathbf{x})\alpha_n(\mathbf{q})\delta\rho_{nq}$$

is that the partial derivatives ∂/∂^j are transformed into coordinates q^j.The gradient $\boldsymbol{\nabla}$ is transformed into a momentum vector $\mathbf{q}$.

Taking the perturbations infinitesimal we do assure, because $\delta\rho_q\delta p_q \approx 0$, that no couplings appear between the Fourier components of different wave numbers.

So the differential equations of the perturbed components transform into linear equations.

In the time between 10^9K and 4500 K, the time of building the Hydrogen-atoms we can take the hydrodynamic limit, where the rate of collisions of photons with free electrons was so great that photons were in local thermal equilibrium with the baryonic plasma

In this limit the wavelength λ of the photon emmited from the fluctuation $\delta\rho_{Rq}$ was greater than the horizon r_H, e.g. outside the time-like area, (the area between the straight lines $r_H = \pm t$). For a detailed description see S. Weinberg. This means that in the case, where $\rho_R$was at the edge of the light conus, $\rho_R + \delta\rho_{Rq}$ could be out of it, Fig.11. 1. In this case there is no communication between the matter ρ_M being in the time-like area and the photon ρ_R being in the space-like area. This violates the logic's causality condition.

However the temperatures of ρ_M and $\rho_R$are measured to be equal.

This was a major problem and the inflation theory was urgently called to solve.

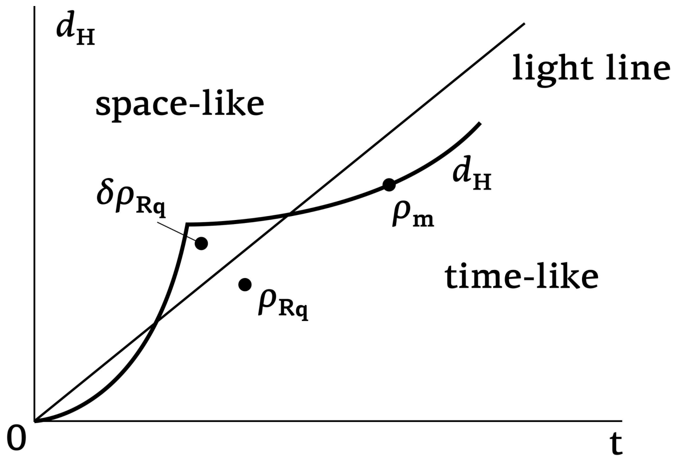

Fig. 11.1

12. THE INFLATION THEORY

12.1. The need of a inflation theory

Inflation gives answers to several problems in Big Bang cosmology that were discovered in the 1970. Inflation was first proposed by Guth in 1979 as the result of a positive energy released from falling the Higgs field (think of it in his dual form as matter) from one unstable to another more stable minimum, Fig.10.1. According to general relativity this fall generates an exponential expansion of space. It was very quickly realised that such an expansion would resolve many other long standing problems. The problems arise from the universe starting from finely tuned initial conditions at the Big Bang and looks like it does today.

The Horizon problem

The horizon problem arises the question why the universe appears statistically homogenous and isotropic. Without the inflation theory and according to the quantum fluctuation theory the mass and the radiation sources are found at not communicating points of the light conus. In this way the causal condition was violated. The inflation theory, taking the radiation source inside the horizon resolved this problem, see Fig. 11.1.

The flatness problem

In the 1960s became known that the density of matter in the universe was comparable to the critical density necessary for a flat universe ($k \approx 0$). What Guth realised was that during inflation H would have been roughly constant, so |k| being the absolute value of k, $\Omega_K = |k|/a^2H^2$ would have been decreasing more or less like a^{-2}. Because $a(t) \sim \exp(Ht)$, at end of the inflation and at the begin of the radiation epoch, we must conclude that Ω_K was negligible. Ω_K was constant negligible also during the whole radiation and the begin of the matter epoch. So the inflation explains the today's flatness problem.

The magnetic -monopole problem

The magnetic monopole problem, sometimes is called exotic -relics problem.

In the Grand Unified Theory at high temperatures (such as it was in the early universe), the electromagnetic force, the weak and strong nuclear forces are not actually fundamental forces but arise due to spontaneous symmetry breaking from a single gauge theory. These theories predict a number of heavy, stable particles that have not been observed in nature. The most notorious is the magnetic monopole, a kind of stable, heavy "charge" of magnetic field. This magnetic monopole has never been found.

With the inflation the space volume got so large and the density so small, that finding a magnetic monopole (if GUT has right foreseen) was a very difficult dask.

These three problems, have the inflation theory successfully resolved. What we know from the early time of the universe is after photon emission.

All before that time is theory that try to validate measured data.

There are many developed inflationary theories, but the most known are the old and the new one.

12.2. The old inflationary theory (Guth's model)

In the vacuum-era $p + \rho = 0$ and there existed nothing but the electromagnetic potential and the Higgs field φ with whose help the energy density and pressure can be written in the form

$$\rho_\varphi = \frac{1}{2}(d\varphi/dt)^2 + V(\varphi), p_\varphi = \frac{1}{2}(d\varphi/dt)^2 - V(\varphi),$$

where $V(\varphi)$ is the potential having a rotational symmetry of φ, sitting on an unstable minimum at the top of a mexican hat, see Fig.10.1.

The energy continuity equation $d\rho/dt + 3H_V(\rho + p) = 0$ takes the form

$$d^2\varphi/dt^2 + 3H_V d\varphi/dt + dV(\varphi)/d\varphi = 0, \qquad (12.1)$$

(we use now the index V instead of Λ),

where $H_V = \dot{a}/a = \sqrt{8\pi G\rho_V/3}$ is the vacuum Hubble constant.

This is the equation of a particle of unit mass with two-dimensional coordinate φ moving in a potential $V(\varphi)$ with a frictional force $-3Hd\varphi/dt$.

The possibility of an early value of $V(\varphi)$ to be constant was it to be sitting on a local minimum of the form

$$V\big(\varphi(t = 0)\big) = -\frac{1}{2}\mu^2\varphi^2 + \frac{\lambda^2\varphi^4}{4} + V(0).$$

The symmetry breaking while falling to a new minimum $\varphi = \mu/\lambda$ causes an exponential growth of the Robertson Walker scale factor $a(t){\sim}\exp(Ht)$ as solution of the Friedmann equation for $k = 0$. The exponential growth followed an expansion like that of today.

The basic idea of the inflationary theory of Guth is that the cosmos will undercool staying at the unstable minimum at $\varphi = 0$ for a time Δt, Fig. 10.1. As the very early universe further cooled at the stabler minimum $\varphi = \mu/\lambda$, transforming his potential energy into kinetic energy of the walls. In the new minimum, through the Higgs mechanism, bubbles are built, which escaped via quantum tunneling. Bubbles at the new minimum form a sea expanding exponentially. Guth recognised that this model was problematic because the bubble sea did not reheat properly. When the bubbles nucleated, they did not generate any radiation. Radiation could only be generated by collisions between bubble walls. But the inflation lasting long enough, the collisions became exceedingly rare to reheat (Hawking).

The region outside the bubble is casually decoupled from the interior of the bubble.

Outside the bubbles the universe underwent an inflation.

The creation and the growth of the bubbles cannot keep up with the inflation of the regions between the bubbles. Then concentrations of bubbles form, which are few bubbles pro domain. This way a very inhomogeneous universe appears, quite in contrast to what we observe. This made necessary to change the old inflationary theory in some points and so we came to the new inflationary theory.

12.3. The new inflationary Theory

Linde 1982, Albrecht and Steinhart 1982 suggested a way, which keeps the good aspects of the old model and changes it so that the theoretically foreseen properties match with the observed ones.

The central idea of this theory is a very special way of symmetry-breaking which results in that the whole observable universe evolved out of a single droplet Fig.12.1.

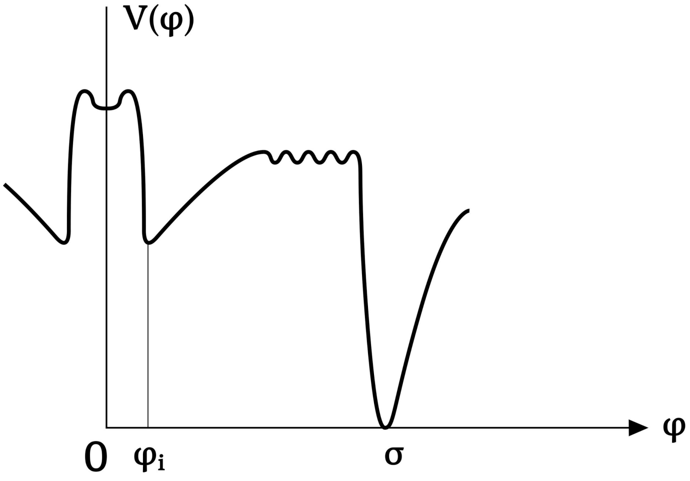

Fig.12.1

The new conception is to have a plunge of the expected mean value ($\langle\varphi\rangle$) of the Higgs field from $\varphi = 0$ to some small initial value $\varphi_i < \sigma$ by quantum mechanical tunneling or through thermal fluctuations.

Starting then from φ_i the Higgs field follows the classical Klein Gordon equation

$$(\partial_\mu\varphi\ \partial^\mu\varphi) = -dV(\varphi)/d\varphi,$$

So the energy continuity equation (6.1.10) takes the form

$$d^2\varphi/dt^2 + 3Hd\varphi/dt = -dV(\varphi)/d\varphi.$$

This equation should be viewed as an equation of expectation values

$$\varphi = \langle\Phi\rangle.$$

For a SU(5) gauge theory the order parameter is a set of scalar fields

$$\Phi^{\alpha}, \alpha = 1, 2, \ldots, 24.$$

There are $5^2 - 1 = 24$ generators,which we call τ_{α} in the fundamental representation. These Φ^{α} are 24 traceless Hermitian 5×5 matrices. One can write down a potential $V(\langle\Phi\rangle)$ for $\langle\Phi\rangle = \langle\Phi^{\alpha}\tau_{\alpha}\rangle$.

This is called Georgi-Glashow model (M. Schwartz)

The number of massless gauge bosons A_{μ}^{α} is determined by the subgroup of SU(5) that is unbroken at this vacuum expectation value. After symmetry breaking we have $SU(5) = SU(3) \otimes SU(2)$.

The gauge symmetry SU(3) consists of three quark fields (u, d, s), which exists in three colour states and the electron, neutrino (τ^{-}, ν_{τ}) and proton, neutron (p, n), which in SU(2) exists in two states. Thus we have the SU(5) matrix

$$SU(5) = \begin{pmatrix} x & x & x & o & o \\ x & x & x & o & o \\ x & x & x & o & o \\ o & o & o & \mathbf{o} & \mathbf{o} \\ o & o & o & \mathbf{o} & \mathbf{o} \end{pmatrix}$$

The SU(3) subgroup on the top left build a 3×3 block of quarks in three colors and the SU(2) subgroup on the bottom right 2×2 block of (τ, ν_{τ}) and (p, n). This SU(5) symmetry is unbroken and each component of it commutes with each other. In order to take a minimum for the potential we must diagonalize the expectation value. This is possible because $\langle\Phi\rangle$ is Hermitian (G.Boerner).

In addition to the block-diagonal SU(3) and SU(2) generators, there is also the generator corresponding to φ, who generates the

U(1) subgroup.

There are two amazing things about this type of grand unification. The gauge coupling constants are related and must equalize at high temperatures and the quantum numbers of the quarks and leptons (generations and colors) are all existent in the SU(5) group.

The potential $V(\varphi)$ at t=0 is taken to be (Coleman and Weinberg 1982)

$$V(\varphi) = \frac{25}{16}\alpha^2(\varphi^4 \ln(\varphi/\sigma) + \frac{1}{2}(\sigma^4 - \varphi^4)), \qquad (12.2)$$

Equation (12.1) for the Higgs field can be coupled to the radiation density $\rho_R = T^4$ (Albrecht et al 1982).

A friction term $\delta = a\alpha^2(d\varphi/dt)^2\varphi, (a = \text{const.})$ is added to the continuity Eq.12.1. Thus

$$d^2\varphi/d\varphi^2 + 3Hd\varphi\,/dt = -dV/dt - \delta, \qquad (12.3)$$

$$H^2 = 8\pi\rho_V G/3. \qquad (12.4)$$

For H the vacuum constant scale provides a reasonable approximation.

The system of equations (12.2), (12.3) and (12.4) can be solved numerically. For the classical evolution with initial values $\varphi(0) \approx \varphi_i \approx 10^8$GeV and $d\varphi/dt = 0$ $(t = 0, \varphi\ = 0$ freezing at the minimum) the solution is shown in Fig. 12.1.

The results show that there is a phase $\varphi = \varphi_i$with stagnate and a phase with exponential expansion $a \sim \exp(Ht)$, (de Sitter model) which lasts until $Ht = 190$ and produces an inflation of $a = e^{190} = 10^{83}$ large enough to achieve all the observed things for that inflation picture.

A bubble of initial size of order $H_V^{-1} = 10^{-26}$ cm is then stretched out to a dimension $H_V^{-1} = \exp(190) = 10^{83}$ cm.

With an estimated radius of the universe (B.F. Schutz) of 10^{28} cm, we live inside an inflated bubble. All the things we can observe comprise just a tiny part of the universe. The entropy, horizon, magnetic monopole problems and flatness are solved, via the inflation.

We can see from Fig.12.1 that φ stays constant ($\varphi = \varphi_i$) for a time and then increases executing damped oscillations with a period

$$\tau_{osc} = 4.8 \times 10^{-4} H_V^{-1}.$$

We have not yet reached the stable minimum $\varphi = \sigma$.

A very critical parameter in the solutions is the starting value φ_i. Only for $\varphi_i \leq 7 \times 10^8$ GeV is sufficient inflation possible. As a consequence a mass density ρ_V produced through the Higgs field has to satisfy the condition $\rho_V \leq \rho_C = 0.92 \times 10^{-26} \text{kg m}^{-3}$.

The inflationary model requires ρ_V to be close to zero, an extrem fine tuning with respect to the GUT scale .

A small effective mass density term would destroy the inflation.

Within the usual GUT theories such mass terms arise naturally from the coupling of the Higgs field φ with the electromagnetic field A^μ of the spacetime.

As a conclusion, neither the old nor the new inflationary theory provides an adequate answer to the observations.

So although every year a new inflationary model is proposed, the problem remains until today unresolved.We must be satisfied only with the observational data.

13. THE RADIATION EPOCH

This epoch ($p=\rho/3$) followed the inflation period. The particles (photons and neutrinos) built through electron- position annihilation to reheat the universe were relativistic and the expansion coefficient

$$a(t)\sim\sqrt{t}, \rho_R\sim a^{-4}\sim t^{-2}, t\sim a^2 \tag{13.1}$$

The deceleration value was $q_0 = 1$ and the Hubble parameter $H = 1/2t,\ H^2\sim\rho_R = T^4$

The temperature at this era began at 10^{10}K and ended at 4500K, at the equality of $\rho_R$with ρ_M.

13.1. Nucleosynthesis

Let us consider what happened to nucleus during this epoch.

The weak interaction allow two bidirectional collision reactions between nucleons and leptons:

$$n + \nu \rightleftharpoons p + e^-, n + e^+ \rightleftharpoons p + \overline{\nu}, \tag{13.2}$$

and a decay reaction in one direction and a three body reaction in the other direction :

$$n \rightleftharpoons p + e^- + \overline{\nu}, \tag{13.3}$$

where $\nu = \nu_e$ the neutrino-partner of the electron; the other flavors (μ, τ) do not contribute at these low temperatures $k_{BT} \ll m_N$ of this epoch. So because of the great difference in the masses of nucleons and leptons the nucleon mass m_N can be treated as essentially at rest. The initial and final lepton energies after the collision are therefore simply related by

$$E_e - E_\nu = Q \text{ for } n + \nu \rightleftharpoons p + e^-,$$

$$E_\nu - E_e = Q \text{ for } n + e^+ \rightleftharpoons p + \bar{\nu},$$

$$E_\nu + E_e = Q \text{ for } n \rightleftharpoons p + e^- + \bar{\nu},$$

where $Q = m_N - m_P = 1.293$ MeV.

a. The reaction 13.2. The total rates $\lambda(p \to n)$ and $\lambda(n \to p)$ are known to the elementary particle physicists and their ratio for a time independent temperature, equal to the neutrino decoupling of the electron T_ν is

$$\lambda(p \to n) / \lambda(n \to p) = \exp(-Q/k_B T_\nu)\,.$$

The ratio X_N of the neutrons to the sum of neutrons and protons, $(X_N + X_P = 1)$ can be calculated from the differential equation

$$dX_N/dt = -\lambda(n \to p)\, X_N + \lambda(p \to n)(1 - X_N).$$

The solution of it that is time independent expressed as

$$X_N/X_P = X_N/(1 - X_N) = \exp(-Q/k_B T)$$

It is the inequality of T and T_ν as the time dependence of the temperature that drives X_N/X_P away of its equilibrium value from Eq.13.2.

For $k_B T \gg Q = 1.293$ MeV we can set $Q = m_e = 0, T_\nu = 0$ and get $\lambda(p \to n) = \lambda(n \to p) = 0.4(T/10^{10}K)^5 sec^{-1}$, (S. Weinberg).

The time $t \approx 1/2H{\sim}T^{-2}$ and the ratio $\lambda/H = 0.8(T/10^{10}K)^3$. At $T = 1.08 \times 10^{10}K, \lambda/H = 1$.

So nuclei are built by a chain of two body reactions: first

$$p + n \to d + \gamma, \text{ then } d + d \to He^3 + p \text{ and } d + d \to He^3 + n$$

and next

$$d + He^3 \rightarrow He^4 + n \text{ and } d + He^3 \rightarrow He^4 + p.$$

The nuclear reaction that built up He^4 from free nucleons (first reaction) at $T \approx 10^9$K was not perfect efficient but left over a small residue of light elements, H^2, H^3, He^3, Li^7 and Be^7. The nuclei of H^3 decayed later by β^+ emission to He^3 and the nuclei of Be^7 decayed later by electron capture to Li^7, leaving us with atoms H^2, He^3, Li^7, He^4 and protons.

A little later than 1.08×10^{10}K at temperatures between 10^{10} and 3×10^9K the two body electron-proton conversion reaction rates became negligible, due partly to disappearance of electron-positron pairs, because of the reheating of the universe.

The conversion of neutrons to protons eventually was ceased by the formation of stable nuclei (He^4).The density at this temperature is too low for any but two body collision reactions to compete with the expansion rate.

b. The reaction (13.3). What remained was the decay of neutrons with a mean life time $\tau = 885.7$ sec, so the neutron fraction became proportional to $\exp(-t/\tau)$.

13.2. Baryon and lepton synthesis

As described in the Sec. 10 about the function of the Higgs mechanism the Higgs field (boson) produces from the electromagnetic potential A^μ electrons, quarks and their antiparticle. The proton is built from three quarks the uud, the neutron from the quarks udd and so on. The meson π^+ from the quarks $u\overline{d}$ and the meson D^0 from the quarks $c\overline{u}$.

The mediators of the weak interaction W^+, $W^-$and Z have also become mass through the Higgs boson. They have been discovered at CERN in 1983, have masses $m_W = 80.4\ \text{GeV}, m_Z = 91.2\ \text{GeV}$ and all three spin 1, as the γ quantum, mediator of the electromagnetism.
As example we consider the inverse muon decay

$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e$$

mediated through the W^- boson.
The positively charged current $j_\mu^+ = \overline{e_L}\gamma_\mu\nu_L$ is mediated through the W^+boson,while the scattering process

$$\overline{\nu_\mu} + e^- \rightarrow \overline{\nu_\mu} + e^-$$

is mediated through the neutral Z boson.
In the cases of leptons, the couplings to $W^\pm$ takes place strictly within a particular of the three generations I, II and III (flavors)

$$(\nu_e, e), (\nu_\mu, \mu), (\nu_\tau, \tau).$$

The corresponding generations (flavors) for the quarks is similar

$$(u, d), (c, s), (t, b).$$

Each quark flavor comes in three colors red,blue and green.
Then a state can be written in a row form

$$\psi = (\psi_{fR}, \psi_{fB}, \psi_{fG}), f = 1,2,3.$$

The binding force between the quarks are eight massless gluons g_i, $i = 1,2, \ldots, 8$

14. MATTER EPOCH

The matter dominated epoch

$$(p = 0), a(t) \sim t^{\frac{2}{3}}, \rho_M \sim a^{-3}, \qquad H = 2/3t.$$

The ending of the radiation era is a virtual one meaning zero existence of other forms of energy. The today's era is a mixture of the bright, dark, radiation and vacuum energy. The dark matter can only be estimated through gravitational lenses. The first matter were electrons, positrons, quarks, and from them protons, neutrons, mesons and the visible H atom. The building of atoms was the end of the darkness epoch, at $t = 380000$ years. Photons began to be emitted from different positions of the cosmos and reached us. The today's matter density is

$$\rho = \rho_\Lambda + \rho_R + \rho_M$$

Where ρ_Λ is the vacuum, ρ_R the radiation and ρ_M the sum of visible (atoms with baryons and electrons) and dark matter, $\rho_M = \rho_B + \rho_D$. Today's measurements give the value $\rho_M \approx 31.7\%$. Further detailed measurements show that $\rho_B \approx 4.9\%$ and $\rho_D \approx 26.8\%$. These values are powerfully reinforced by observations of the anisotropies in the cosmic microwave background Fig. 21.2, (G. Boerner). If the dark matter is so dominant, arises the claim to know the properties and its constituents.

We know that this matter is dark, in the sense that it does not act significantly with radiation,both because we don't see it, but also because it has not lost its kinetic energy sufficiently to relax into the discs of galaxies, as the baryonic matter. This means in particular that these particles must be electrically neutral. Detailed studies of the dynamics of the galaxy clusters indicate that the dark matter particles must also be cold, in the sence that their velocities are highly non-relativistic.

Following S. Weinberg (the study of a double galaxy 1 E0657-558 (the “bullet cluster”) with $z = 0.296$ has provided vivid direct evidence of the existence of dark matter, which has gravitational interactions with itself or with ordinary baryonic matter. The galaxies of this cluster are mostly grouped into two distinct subclusters, while hot gas (observed through its emmision of X-rays) is concentrated between these subclusters. The interpretation is that two clusters of galaxies have collided; the galaxies which have little chance of close encounters have been attracted and going through each other without interaction continued on their original paths, while the two clouds of hot gas that previously accompanied them have collided and were encircled and stabilized because of the cosmological constant Λ in form of the pressure of the hot gas to the center of the double cluster. The ratio of the mass in hot gas to the mass in all matter is estimated to be about $1/6$, in line with the value Ω_B/Ω_M previously inferred from measurements of deuterium abundance and luminosity distance as a function of redshift, or from anisotropies in the cosmic microwave background).

The pressure of the hot gas had canceled the attraction between the two subclusters of the dark matter, holding them in distance. It is found that most of the dark matter is not associated with the hot gas. Any dark matter component is a downgrading free gas, while dissipative effects can convert the kinetic energy of baryonic matter into radiation. Radiative cooling then allows the baryonic matter to sink to the center of the collapsed configuration, while the initially well-mixed, dark matter component separates and form an extended halo around the central condensation.

If the dissipative collapse of the baryonic matter is halted by angular momentum, a disc is formed. If star formation stops, the dissipative collapse of baryons, form a spherical system.

The total matter density is mapped out through its effect in gravitationally deflecting light from more distant galaxies along the same line of light, (see Sec. 23).

Elementary particle theory offers several candidates for the particles of the cold dark matter. A great goal is to determine these particles.

15. PROCESSES DURING THE TEMPERATURE FALL

After the time zero of the Big -Bang the temperature began to fall. At symmetry breaking the temperature is estimated to be 10^{12}K.

At the inflation period the universe undercooled. At 10^{12} to 10^{11} appeared the τ fermions with mass in equilibrium with the left neutrinos ν_τ and the $\overline{\tau}$ antifermions with mass in equilibrium with the right antineutrinos $\overline{\nu_\tau}$. Later appeared the μ fermions and $\overline{\mu}$ antifermions with mass and in equilibrium with the neutrinos ν_μ and $\overline{\nu_\mu}$ with the same helicity as the τ generation. At 10^{10}K began the decoupling of the left neutrinos ν_e of the electrons. Later at $T \approx 10^9$ to 10^{10}K began the reheating of the universe through fermion-antifermion annihilation. The radiation era began at $T \approx 10^8$K until the equilibrium of radiation and matter energy density at 4000 K.

The building of H-atoms began at 4500 K. At 3000 K was the last scattering of photons to arrive at us as microwave of 13 mm wavelength.

Now we will look back when the temperature was between $10^4$and 10^{11}K which is low enough so that $\tau - \overline{\tau}$, muon-antimuon and hadron-antihadron pairs were not longer being produced .

Following (S. Weinberg), (there are two circumstances that greatly simplify this task. The first is that the collision rate of photons and electrons and other charged particles during this era was so much greater than the expansion rate of the universe that the photons and charged particles can be assumed to have been in thermal equilibrium, with a common falling temperature. At significantly early times even the neutrinos and perhaps the cold dark matter particles were also in thermal equilibrium with the photons and charged particles. Later when, they no longer collided rapidly with other particles, they can be treated separately as free particles. The other circumstance is that the number density of baryons is so much less than the number density of photons that we can ignore the chemical potential associated with the baryon number. Also because the electron/photon number ratio is so small now, it is reasonable to assume that the universe has always had a very small net lepton number density per photon. This means that even at temperatures of order 10^{10}K the energy density, the pressure and the entropy density were functions of the temperature alone.

To have an insight of the thermal history is necessary to look at the thermodynamics and statistical mechanics of this sort of matter, in thermal equilibrium with negligible chemical potentials.

The condition of thermal equilibrium tells us that the entropy in a co-moving volume is fixed

$$s(T)a^3 = \text{constant}.$$

The second law of thermodynamics says that any heat change dQ in a system of volume V produces a change in the entropy given by

$$dS = dQ/T = (dU + p(T)dV)/T,$$

where $S = s(T)V$ and $U = \rho(T)V$.

Equating the coefficients of dV gives our formula of the entropy density

$$s(T) = [\rho(T) + p(T)]/T \tag{15.1}$$

Also equating the coefficients of VdT and using Eq.15.1 give the law of conservation of energy

$$\mathrm{Tdp(T)/dT} = \rho(\mathrm{T}) + \mathrm{p(T)}.$$

With equal numbers of particles and antiparticles the number density n(p)dp of fermions (such as electrons) or bosons (like photons) of mass m and momentum between p and p + dp is given by the Fermi-Dirac or Bose-Einstein distributions (with zero chemical potential)

$$\mathrm{n(p,T)} = [4\pi \mathrm{gp}^2/(\mathrm{h}^3]\mathrm{x}[1/(\exp(\sqrt{\mathrm{p}^2+\mathrm{m}^2}/\mathrm{k_B T}) \pm 1]$$

where g is the number of spin states of the particle and antiparticle and the sign is + for fermions and − for bosons. For instance, for photons g = 2, because photons have spins ±1 and they are their own antiparticles, while for electrons and positrons g = 4, because they have two spin states ±½ and electrons and positrons are distinct particles. The energy density and pressure of a particle of mass m are given by the integrals

$$\begin{aligned} \rho(\mathrm{T}) &= \int_0^\infty \mathrm{n(p,T)}\,\mathrm{dp}\sqrt{\mathrm{p}^2+\mathrm{m}^2}, \\ \mathrm{p(T)} &= \int_0^\infty \mathrm{n(p,T)}\,\mathrm{dpp}^2/3\sqrt{\mathrm{p}^2+\mathrm{m}^2}. \end{aligned} \tag{15.2}$$

The entropy density of this particle is then given by the Eq.15.1 as

$$\mathrm{s(T)} = \frac{1}{\mathrm{T}}\mathrm{n(p,T)dp}[\sqrt{\mathrm{p}^2+\mathrm{m}^2} + \mathrm{p}^2/3\sqrt{\mathrm{p}^2+\mathrm{m}^2}] \tag{15.3}$$

In particular, for massless particles Eq.15.2 gives

$$\rho(T) = g\int_0^\infty 4\pi p^3\, dp h^{-3}[1/\exp(p/k_B T) \pm 1] =$$

$= g a_B T^4/2$ for bosons and

$= 7 g a_B T^4/16$ for fermions $= 7/8[\rho(T)]$ for bosons.

At this radiation era $p(T) = \rho(T)/3$ and $s(T) = 4\rho(T)/3T$.

During this period of thermal equilibriun the temperature as function of the time is governed by the Eq.15.1 and the Friedmann Eq.6.2.4 with $k = \Lambda = 0$

$$\dot{a}^2/a^2 = 8\pi G\rho(t)/3.$$

Combining these and integrate gives

$$t = -\int (ds/dT)\, dT/(s(T)\sqrt{24\pi G\rho(T)}) + \text{constant} \qquad (15.4)$$

With relativistic particles and $m = 0$ the entropy and energy density are given by

$$s(T) = 2Na_B T^3/3$$

$$\rho(T) = Na_B T^4/2$$

for bosons and fermions,N is the particle types counting particles and antiparticles and each spin state separately and with an extra factor 7/8 for fermions.

The time as function of the temperature was

$$t = \sqrt{3/16\pi G N a_B} T^{-2} + \text{constant}, \qquad (15.5)$$

where N is the number of particles and antiparticles for bosons and fermions and $a_B$the radiation energy constant).

Now start at a time when the temperature was around 10^{11} K, which is in the range $m_\mu \gg k_B \gg m_e \gg$. Even though it was too cold at this time for reactions of the τ or μ leptons like $\nu_\mu + e \rightarrow \mu + \nu_e$ or $\nu_\tau + e \rightarrow \tau + \nu_e$, the μ and τ neutrinos and antineutrinos were kept in thermal equilibrium by neutral current reactions, like neutrino-electron scattering or annihilations $e^+ + e^- \leftrightarrow \nu + \bar{\nu}$. Hence the constituents of the universe at this time were photons with two spin states plus three species of neutrinos and antineutrinos, each one (left for ν, right for $\bar{\nu}$) spin state, plus electrons and antielectrons (positrons), each with two spin states, all in equilibrium and all highly relativistic, giving

$$N = 2 + 7(6 + 4)/8 = 43/4.$$

So that Eq. 15.1 gives in cgs units

$$t = 0.994 \sec(T/10^{10}K)^{-2} + \text{constant}.$$

For instance, with muons ignored and the mass of electrons neglected,it took 0.0098 sec for the temperature to drop from a value 10^{12}K to 10^{11}K, another 0.998 sec to drop to 10^{10}K and another 167 sec to drop to 10^9K.

16. THE COSMOLOGICAL REDSHIFT

Imagine a ray of light emitted at a distance distance r from the earth (index e) coming to us along the z-axis ($\theta = \varphi = 0$).
In spherical coordinates and a Robertson- Walker metric

$$ds^2 = 0, dt = -a(t)dr/\sqrt{1 - kr^2}.$$

Writing $\Delta t_0 = a(t_0)\, \Delta r/\sqrt{1 - kr^2}$ and $\Delta t_E = a(t_E)\, \Delta r/\sqrt{1 - kr^2}$ and divide them one takes

$$\Delta t_0/\Delta t_E = a(t_0)/a(t_E) = \lambda_0/\lambda_E\ .$$

Putting $\Delta t_0 = T_0$, $\Delta t_E = T_E$ the period of the light wave at the earth and the emitting star, one takes

$$T_0/T_E = a(t_0)/a(t_E) = \lambda_0/\lambda_E\ .$$

where λ_0, λ_E are the wavelengths of the ray at the earth and at the star respectively. The shift follows from $\lambda_0/\lambda_E = 1 + z$, z is measured positive, so $a(t_0)/a(t_E) > 1$
and because $t_E < t_0$ we conclude that the universe expands. The distance r of the star was $a(t_E)r$ and now this is $a(t_0)r$.
At the preceding time t_E the matter density was greater than today because of the expansion

$$\rho(t_E) = \rho_V + \rho_R x^{-4} + \rho_M x^{-3},$$

where

$$x = a(t_E)/a(t_0) = 1/(1 + z) \tag{16.1}$$

17. THE TIME VARYING VACUUM ENERGY DENSITY (QUINTESSENCE)

The natural way to introduce a varying vacuum energy density is to assume the existence of one or more scalar fields that the vacuum energy density depends on and whose expectation values vary with time. Such fields are introduced in inflation theories. We are interested here in the case of Robertson-Walker metric in a scalar field that depends only on time, not position. In this case the formulas for the scalar field energy density and pressure become

$$\rho_\varphi = \frac{1}{2}(d\varphi/dt)^2 + V(\varphi)$$

$$p_\varphi = \frac{1}{2}(d\varphi/dt)^2 - V(\varphi)$$

As a result $(1 + w)\,\rho_\varphi \geq 0$,where $w = p_\varphi/\rho_\varphi$. So the case $w = -1$ represents the time $t = 0$ of the Big Bang.We will now consider the time $t > 0$. The Eq.6.1.10 of energy continuity here yields

$$d^2\varphi/dt^2 + 3Hd\varphi/dt + dV(\varphi)/dt = 0. \qquad (17.1)$$

This is the equation of motion of a particle of unit mass with one dimensional coordinate φ moving in a potential $V(\varphi)$ with a friction force $-3Hd\varphi/dt$. The field will run toward lower values of $V(\varphi)$, finally coming to rest at a local minimum of $V(\varphi)$. We can adjust an additive constant in order to make it vanish at the minimum.

- The original and simple example is provided by a potential

$$V(\varphi) = M^{4+\alpha}\varphi^{-\alpha}, \tag{17.2}$$

where α is a constant $0 < \alpha < 1$ and M also a constant, which gives $V(\varphi)$ the dimension of energy density. Following S. Weinberg,for any potential it is necessary that at sufficiently early times ρ_φ was much less than the energy density ρ_R of radiation, because any appreciable increase in the energy density at the time of nucleosynthesis, would lead to a He^4 abundance exceeding what is observed. At these early times ρ_R is also greater than ρ_M.

Eq.13.1 gives $a_R{\sim}t^{1/2}$ and $H = 1/2t$. The field equation (17.1) with the potential (17.2) then yield

$$d^2\varphi/dt^2 + 3(d\varphi/dt)/(2t) - \alpha M^{4+\alpha}\varphi^{-\alpha-1} = 0 \tag{17.3}$$

This has a solution

$$\varphi = \{[\alpha(2+\alpha)^2 M^{4+\alpha}t^2]/(6+\alpha)\}^{1/(2+\alpha)} \tag{17.4}$$

Both $(d\varphi/dt)^2$and $V(\varphi)$ then go as $t^{-2\alpha/(2+\alpha)}$ and therefore at very early times $\rho_\varphi = \rho_V$ must have been less than ρ_R which goes as t^{-2}, Fig.17.1. The solution Eq.17.4 is not unique, but is an attractor in the sense that any other solution that comes close to it, will approach it as time increases. To see this, note that a small perturbation $\delta\varphi$ of the solution $\overline{\varphi}$ of (17.3) and $\varphi = \overline{\varphi} + \delta\varphi$ (Eq. 17.3) yields

$$d^2\delta\varphi/dt^2 + 3(d\delta\varphi/dt/dt)/(2t) + \alpha(1+\alpha)M^{4+\alpha}\varphi^{-\alpha-2}\delta\varphi = 0.$$

This has two independent solutions of the form

$$\delta\varphi \sim t^{\gamma}, \gamma = -\frac{1}{4} \pm \sqrt{1/16 - (6+\alpha)(1+\alpha)/(2+\alpha)^2}.$$

The square root is imaginary for $\alpha > 0$,so both solutions for $\delta\varphi$ decay as $t^{-1/4}$ for increasing t .For this reason, the particular solution $\overline{\varphi}$ of Eq.17.3 that goes as Eq.17.4 is known as the "tracker solution", Fig.17.1.

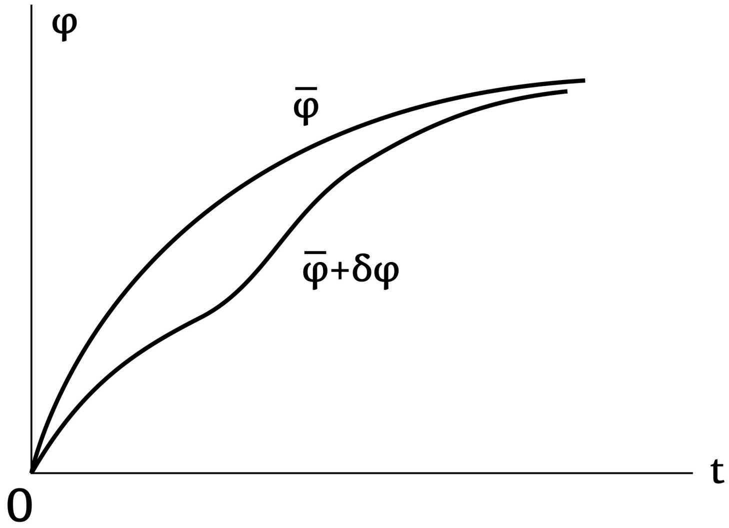

Fig.17.1

The energy densities are ρ_{V0} the today's $(t = t_0)$ and ρ_V at an earlier time t, $z \sim 1/t$.

Nothing much changes when the radiation energy density drops below ρ_M.The tracker solution $\overline{\varphi}$ continues to grow as $t^{2/(2+\alpha)}$ and $(d\varphi/dt)^2$ and $V(\varphi)$ as $t^{-2\alpha/(2+\alpha)}$. But ρ_M and ρ_R are decreasing faster, like t^{-2}, so eventually ρ_M and ρ_R will fall below ρ_V.

- Similar results are produced if we take as potential the" linear" form

$$V(\varphi) = V_0 + V'_0(\varphi - \varphi_0).$$

The today's $w = p_\varphi/\rho_\varphi$ is estimated to be $w = -0.777$.

The values of ρ_V, ρ_{V0} and the redshift $z \sim 1/t$ is shown in Fig.17.2.

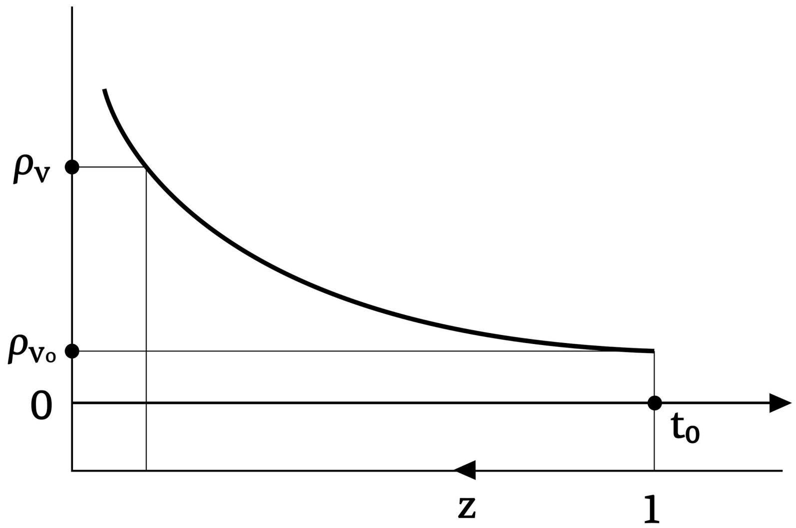

Fig.17.2

We infer from this Figure that the dark energy density at earlier times was greater than today.The explanation result is that $\rho_{VE} = \rho_{BME} + \rho_{DME}$ was decreased, because as his ingredient dark energy density ρ_{DME} had no interaction with the bright matter,had also no friction with it, (the coefficient $3Hd\varphi/dt$ in Eq.17.1 was negligible). So the dark matter was expanding more quickly than the bright matter and was out of the horizon, not contributing longer to the energy density.

Some z and ρ_V/ρ_{V0} values are:

z	tracker ρ_V/ρ_{V0}	linear ρ_V/ρ_{V0}
0	1	1
0.5	1.347	1.2
1	1.712	1.273
3	3.224	1.331
≥1	≥1	1.340

18. THE CALCULATION OF THE AGE OF OUR UNIVERSE

The proper distance at time t_Z from the origin to a comoving star at radial coordinate r according to Sect.6.1 is

$$d(r,t) = a(t_Z)\int_0^r dr/\sqrt{1-kr^2} = a(t_Z)\begin{cases}\sin^{-1} r \text{ for } k=1\\ r \text{ for } k=0\\ \sinh^{-1} r \text{ for } k=-1\end{cases}$$

We define $x(t) = a(t)/a(t_0), a(t_0) = 1, x(t) = a(t)$.

Then $dx/dt = da(t)/a(t_0)dt = (da(t)/dt)/a(t))(a(t)/a(t_0)) = H(t)x(t)$ or because of (6.15)

$$dt = dx/x(t)H(t) = dx/a(t)H_0 =$$

$$dx/x(t)H_0\sqrt{\Omega_V + \Omega_K x^{-2} + \Omega_M x^{-3} + \Omega_R x^{-4}}$$

If we define the zero of time as corresponding to an infinite z, the todays time $t_o = t(z)$, the $x(0) = 0$ and the todays $x = 1/(1+z)$, and integrate we have

$$t(z) = H_0^{-1}\int_0^{1/(1+z)} dx/\sqrt{\Omega_2},$$

$$\text{where } \Omega_2 = \Omega_V x^2 + \Omega_K + \Omega_M x^{-1} + \Omega_R x^{-2}\,.$$

Settling $z = 0$ we take the present age t_0 of the cosmos

$$t_0 = H_0^{-1}\int_0^1 dx/\sqrt{\Omega_2} \tag{18.1}$$

For $\Omega_V = 0.72$, $\Omega_M = 0.28$, $\Omega_K = \Omega_R = 0$ and $H_0 = 70$ km/sec the above integral gives $t_0 = 13.4 \times 10^9$yrs.

19. THE LUMINOSITY AND MEASURING OF THE UNIVERSE'S AGE

The most usual method of determining distances in astronomy was the determination of the apparent luminosity. The absolute luminosity of a light source is defined to be the emitted energy per second and the apparent luminosity at a distance d is the energy pro second and quadrat centimeter arrived.

That is

$$l = L/(4\pi d^2)\ \mathrm{erg cm^{-2} sec^{-1}}.$$

At large distances this formula needs to be modified for three reasons.

- At the time t_0 that the light reaches the earth the proper distance r from the star to the earth becomes $a(t_0)r$. Thus the sphere surface $4\pi d^2$ must be replaced by $4\pi[a(t_0)r]^2$.
- The frequency of the photon arriving on earth is decreased from the frequency emitted from the star by the redshift factor $1/(1+z)$.
- The energy of the photon arriving on earth is decreased from the energy emmited from the star by the redshift factor $1/(1+z)$.

Putting these together we have $l = L/(4\pi d_L^2)$,where $d_L = a(t_0)d(1+z)$.

In the second century AD astronomer Claudius Ptolemy published a catalog of 1022 stars with luminosity m belonging to one of 6 categories. Stars with m = 1 were bright stars and stars with m= 6 were stars barely visible.

In 1856 Norman Pogson made the discrete classification of the Ptolemy list, continuous, by the formula

$$l = 10^{-2m/5} \times 2.52 \times 10^{5} \mathrm{erg.\,cm^{-2} sec^{-1}} = L/(4\pi d_L^2),$$

with m a function of d_L.

Later it is decreed that the distance d be d= 10pc, $(1\mathrm{pc} = 3.1 \times 10^{18}\mathrm{cm})$.

So

$$L = 10^{-2M/5} \times 3.02 \times 10^{35} \mathrm{erg.\,sec^{-1}} = L/(4\pi d_L^2),$$

where M is a function $M(1 + z, a_0)$, where L is the absolute and M the apparent bolometric luminosity.

The distance d can also be expressed through their modulus m-M.

$$d = 10^{1+(m-M)/5}\ \mathrm{pc}\,.$$

We can compare the photometric measured distance d as function of the luminosity $m - M$ and the calculated distances from (18.1) as function of z and Ω`s.

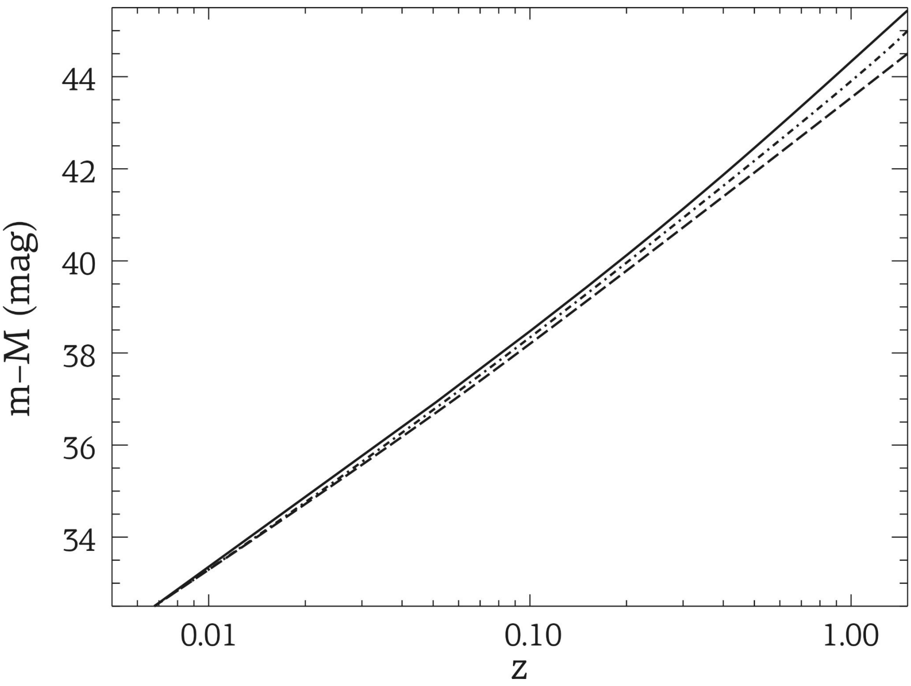

Fig. 19.1

Fig. 19.1 shows with the solid and dashed curves the calculated distances. The photometrically measured distances from A.G. Riess et al., Astron. J 116, 1009 (1998) for some stars match best to the combination $\Omega_\Lambda = 0.76$, $\Omega_M = 0.24$, $\Omega_R = \Omega_K = k = 0$ for the calculated distances of the —— line. The other lines represent:

---- $\Omega_M = 0.20, \Omega_\Lambda = 0, \Omega_K = 0.80,$

while -·-·-·- $\Omega_M = 1.0, \Omega_\Lambda = 0, \Omega_K = 0.$

20. THE STAR EVOLUTION

The evolution of a star takes a very long time, so it is impossible to observe it.

However we can interpret their distance order as time order.One such index is the luminosity and one other is the maximum wave length as it is in the diagram of the Plancks radiation law of the black body. So we come up to the Herzsprung-Russel diagram, Fig.20.1

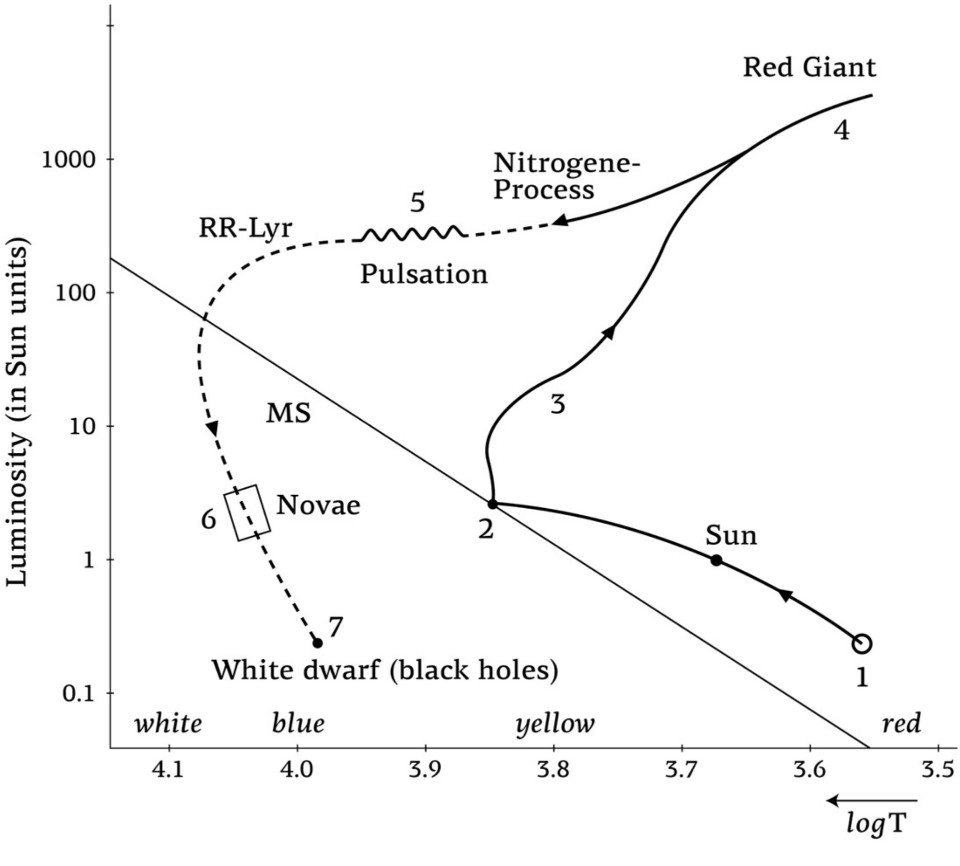

Fig. 20.1

In this diagram we take as abscissa x the temperature from great to lower grades or from blue-green-yellow to red according to the astronomers and as ordinate y the luminosity in units of the Sun luminosity.

We begin with the birth as a red star at point (1). The star grows up moving along a curve undergoing a gravitative contraction to a point (2) at the Main Sequence (MS). In this curve is our Sun, almost in the middle of his life. The place on the main sequence is a function of the mass of the star. The massiver the star, the higher in the MS will he be landed. When the inner temperature reaches $20 \times 10^6 K$, the proton- proton- and the carbon- process begins. These processes are in equilibrium with the radiation emitted. Just before the core is 12% of the mass, the star undergoing a core contraction moves along the curve (3), (a motion computed accurately by Schwarzschild) to land at point (4) as red Giant with temperature $T = 2 \times 10^8 K$. At this phase the nitrogen-process begins, where a carbon core is produced, in an environment of helium, similarly to the helium core in the environment of hydrogen. In the helium zone, helium is transformed in carbon, oxygen, silicon and iron. The first part of the way (4) to (5) is also accurately computed. The rest of the way until the end is based only on observations. At point (5) the star begins eventually to pulse. At point (6) lie the Novae and the Supernovae, who explode increasing their luminosity 15 to 20 times. If the mass of the star is 20 times greater than the Sun's mass what remains after the explosion is enough to begin a new process. The gravitational force is enormous and gives rise to attract continuously mass. For every star there is a critical radius in relation to its mass, where the space-time is so distorted, that it is separated from its environment and acts as a black hole.

Because of the great gravitation force a photon, with its dual property as mass $h\nu/c^2$, can't escape the black hole. Only in the edge of the Schwarzschild radius, after exact measuring of the photon momentum, Δp is very small, and because of the Heisenberg relation $\Delta p . \Delta\, x \approx$ ħ, can Δx be great enough to escape the black hole, as S. Hawking has deduced.

If the mass of the star is less than 20 times the mass of the Sun, through explosions so much mass escapes that it becomes a white dwarf. The white dwarf continues to radiate till his color changes from white to yellow, to red, to violet till he becomes a black dwarf and disappears.

21. THE MICROWAVE RADIATION BACKGROUND

Fig. 21.1 shows the spectrum of the black-body, where the intensity over all wavelengths is proportionate to T^4.

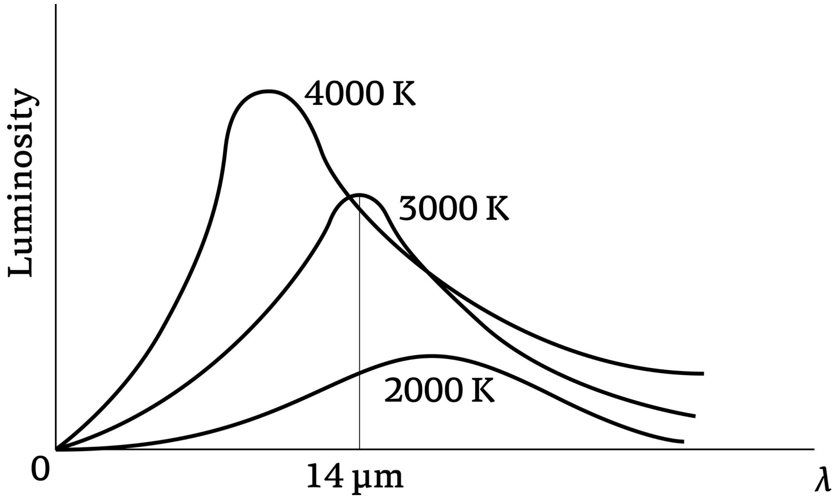

Fig.21.1

The black body can be imagined as a hidden fire at the universe. We see that at the maximum intensity the emitted wavelength λ_E is a function of the temperature T_E

This relation has the hyperbolic form

$$\lambda_E T_E = const. = \lambda_0 T_0.$$

The wavelenght λ_0 reached us, because the redshift satisfies the relation

$$\lambda_0/\lambda_E = 1 + z\ .$$

The measured $\lambda_0 = 13\ mm$ corresponds to $T = 2.725\ K$ and is in the

microwave region.
From the measured $1 + z = 1100$ we conclude that $\lambda_E = 14\,\mu m$ and $T_E = 3000\,K$.
This emission took place long after the Big-Bang.

The temperature 3000 K of the blackbody corresponds to the last scattering of photons to the electrons. The black- body spectrum for the number density of photons in equilibrium with the matter at temperature T and frequency between

$$\nu_T \text{ and } \nu_T + d\nu_T \text{ is } n(\nu_T)d\,\nu_T = 8\pi\nu_T^2\, d\nu_T e^{-1},$$

$$\text{where } e = exp((h\nu_T/k_B T) - 1).$$

In an isotropic and homogenous empty ($K \approx 0$) space $x^\mu = (t, \boldsymbol{x})$ is Lorentz covariant, hν_T/k_BT, (h=Planck-, k_B=Boltzmann constant) being scalar remains constant.
That is $h\nu_\varepsilon/k_B T_E = h\nu_0/k_B T_0$. It follows that

$$\nu_E/T_E = \nu_0/T_0, \text{ or } \nu_E/\nu_0 = \lambda_0/\lambda_E = 1 + z = a_0/a_E = T_E/T_0,$$

$$T_0 = a_E T_E/a_0, \text{ or } T_0 \sim a_0^{-1}, \text{ or } \mathrm{T(t)} \sim a(t)^{-1}.$$

Anisotropies in the cosmic microwave background can occur, if in the light line are interposed galaxies of dark matter, unseen from earth, but still reflecting light because of gravitational interaction with the photons, (see Sec. 23).

Another reason is the spectrum of density fluctuations during the birth of stars.

The mean square temperature fluctuations of the CMB are expected to depend on angle, showing a sequence of peaks at different angles, Fig. 21.2.

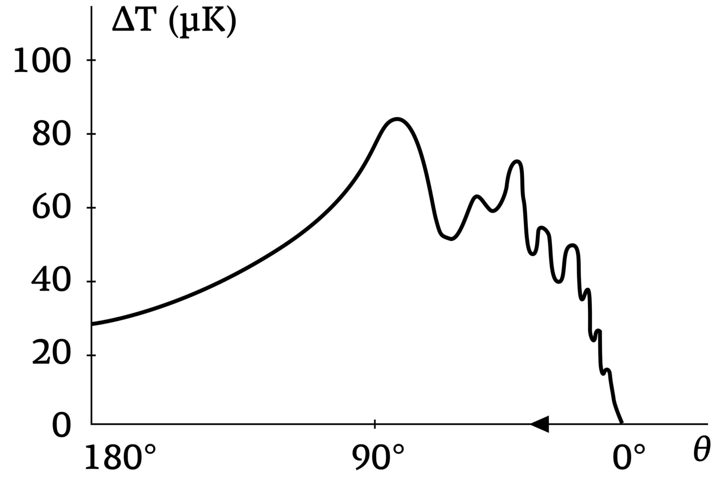

Fig.21.2

22. AGES ESTIMATED THROUGH NUCLEUS DECAYS

Apart of the estimation of the cosmos age t_0 from the equation (18.1), we can think to estimate t_0 using the natural decay of an isotope. If the initial concentration is c_0 and the decay rate λ, the today concentration is

$$c = c_0 exp(-\lambda t) \qquad (22.1)$$

We go further if we use two isotopes with the relative present concentration c_1/c_2
which is

$$c_1/c_2 = exp[(\lambda_2 - \lambda_1)t]c_{10}/c_{20}, \qquad (22.2)$$

with c_{10}, c_{20} the initial abundances of the radioactive elements $E_1$and E_2.

Abundance deductions of certain long-lived radioactive isotopes can be employed as chronometers to determine the ages of the oldest stars.

There have been a number of recent detections of the element Th, with a half-life of 14 Gyr, in the poor Fe halo stars. This element, along with Uranium, is synthesised solely in the rapid neutron capture process (r-process).

Comparison of the observed stellar abundance of this radioactive element with its initial (time-zero) abundance in an r-process site leads to a direct radioactive-age estimate of the star. We show in Fig.

22.1 the abundance distribution, including Th, in the star CS 22892-052. While the heavy n-capture elements are consistent with the scaled solar r-process curve, the observed Th abundance lies below this same line.

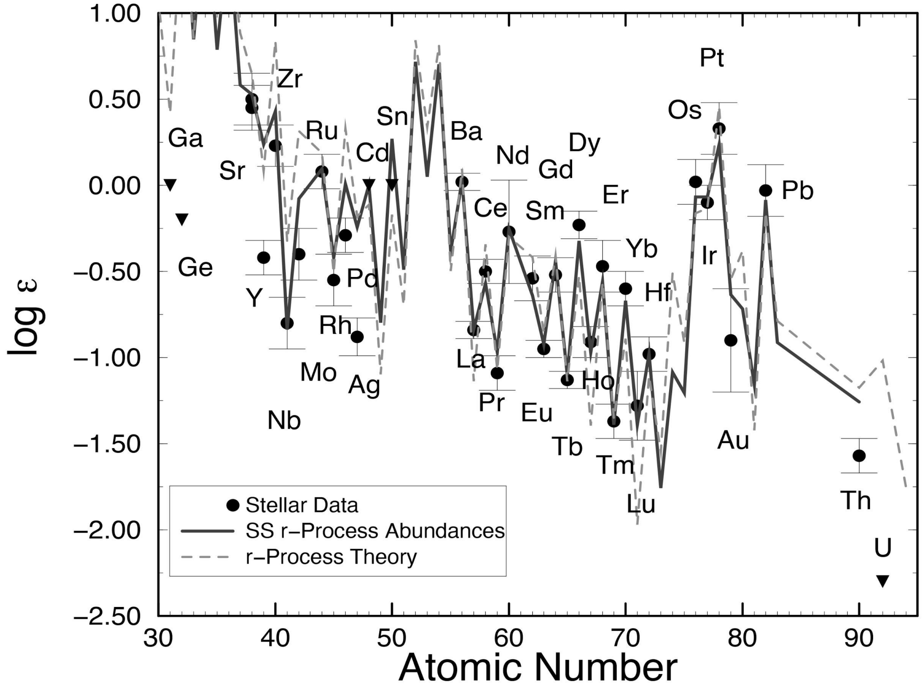

Fig.22.1. Abudances of elements produced by the r-process in the star BD+17^{0}3248, obtained by ground based and Hubble Space Telescope spectroscopic observations. The solid curve gives theoretical initial abundances, based on solar system data. From J.J. Cowan et al., Astrophys. J 572, 861 (2002) [astro-ph/0202429].

This difference is a clear demonstration that this star is older than the sun. To determine how much older requires knowledge of the initial Th abundance that must be predicted from r-process models. Such a model calculation is illustrated in Fig. 22.1 by the dashed line.

Such predictions employ the ratio of Th to another r-process element, typically the Eu.

Going back to Eq.22.1 we have the time

$$t = ln(c_1 c_{20}/c_{10} c_2)/(\lambda_2 - \lambda_1).$$

Setting $A = c_1/c_2$, $A_0 = c_{10}/c_{20}$, $c_{10} = log\varepsilon_{10}$, $c_{20} = log\varepsilon_{20}$ we get

$$t = (logA - log\varepsilon_{10} + log\varepsilon_{20})/(\lambda_2 - \lambda_1). \qquad (22.3)$$

The today's values A, λ_1, λ_2 can be measured and the values of the logε_{10}, logε_{20}can be read from the Fig. 22.1 .

For the pair U^{238}, Th^{232}, Eq. 22.3 gives $t_0 \approx 15.5$ Gyrs, while for the pair Th^{232}, Eu^{152},Eq.22.3 gives $t_0 \approx 11 - 15 \pm 4$ Gyrs.

23. INTERACTION OF THE PHOTON WITH MATTER AND MEASURING OF THE DARK MATTER

23.1 Attraction of the photon from the earth.

One γ-quantum has the energy $E = \omega_0$ and the momentum k ($\hbar = c = 1$). Emitted from an excited atom energy level E_E with middle life time τ goes to the stable ground state E_K.His energy is $\omega_0 = E_E - E_K$, (H. Wegener, R. Moessbauer).

The corresponding middle energy width Γ according to Heisenberg is

$$\Gamma\tau = 1.$$

The nucleus having a momentum vector $\mathbf{p} = M\mathbf{v}$ before the emission has the energy

$$E_1 = E_E + p^2/(2M)$$

and after the emission of the γ quantum with momentum $\mathbf{k}$ the energy is

$$E_2 = E_K + (\mathbf{q}^2)/(2M), \text{ where } \mathbf{q} = \mathbf{p} - \mathbf{k}\,.$$

The energy difference goes to the γ-quantum.

$$\omega = E_1 - E_2 = E_E - E_K - kv\cos\varphi - k^2/(2M) =$$

$$\omega_0 - kv\cos\varphi - k^2/(2M).$$

$kv\cos\varphi$ is the Doppler shift and $k^2/(2M)$ the back push energy.

For the isotope Fe_{26}^{57} is $k^2/(2M) \approx kv = 2 \times 10^{-2}$eV (for a middle room temperature velocity $v \approx 100$ m/s, $\Gamma = 4.6 \times 10^{-9}$eV).

If the Fe_{26}^{57} atom is embedded in a crystal ($v \approx 0$ and $M \approx \infty$ is the crystal mass) then we must substitute $kv\cos\varphi - k^2/(2M)$ through the oscillations energy $\Delta E_{C.}$

$$\text{So} \qquad \omega = \omega_0 - \Delta E_C.$$

The intensity $I(\omega)$ as function of ω has now the form of Fig. 23.1.

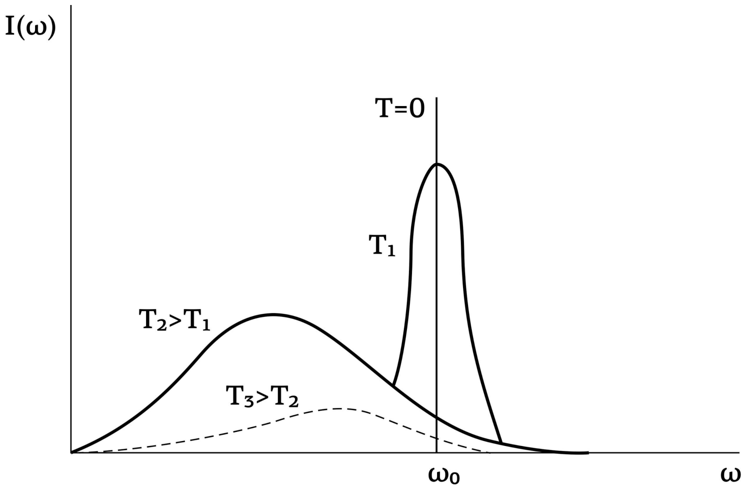

Fig. 23.1

The "line ω_0" (after embedding Fe in a crystal) is named after its discoverer R. Mössbauer, for which he was in the 1961 Nobel laureate.

Using the dual property of the photon as mass, one γ-quantum with energy $\hbar\omega$ has mass $m_\gamma = \hbar\omega_0/c^2$.

In the gravity field of the earth in a high z the potential energy is

$$E_{pot} = m_\gamma gz = \hbar\omega_0 gz/c^2, \, (g = 981 \text{ cm/s}).$$

From the energy conservation we have

$$\hbar\omega_0 = \hbar\omega + E_{pot} = \hbar\omega_0(1 + gz/c^2).$$

This means a redshift ω

$$\Delta\omega = \omega_0 - \omega = gz\omega/c^2.$$

For $z = 22.5$ m $\Delta\omega/\omega = 2.46 \times 10^{-15}$. For the Fe isotope γ-ray this is a shift of $\approx 1\%$ of Γ/ω_0. This is small but with the sharp "Mössbauer line" it is measurable.

Trough undercooling the middle life τ increases and Γ decreases making the ω_0 very sharp.

23.2 Measuring of the dark matter

In the presence of dark matter M a sphere of radius R, functions as a lens.

The radius R of the lens is the closest approach to a light beam and then the deflection is maximized. From the general theory of relativity we know that the deflection angle is

$$\gamma = 4MG/R.$$

For the angles in the Fig.23.2 we get

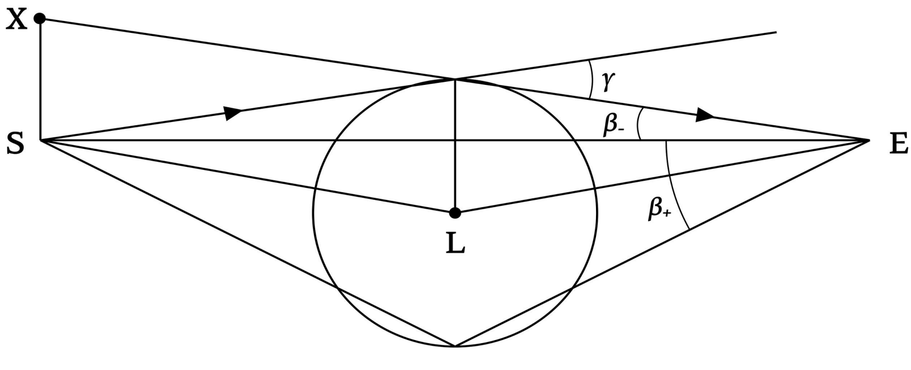

Fig.23.2

$$\beta = R/d(EL), \;\; \gamma = d(XS)/d(LS), \;\; \beta - \alpha = d(XS)/d(ES)$$

From the above equations follows that

$$(\beta - \alpha)\beta = \gamma d(LS). R/(d(ES). d(EL)) =$$

$$4MG.\, d(LS)/(d(ES)d(EL)) = \beta_E^2$$

The equation $\beta^2 - \alpha\beta - \beta_E^2 = 0$ has the solutions

$$\beta\pm = (\alpha/2) \pm \sqrt{\beta_E^2 + \alpha^2/4} \tag{23.1}$$

The angle α is unknown because L is dark but we can eventually measure the angle $\beta_+ - \beta_-$ between the two images at the opposite radii. So the absolute difference

$$\delta = \mathrm{I}\beta_+ - \beta_-\mathrm{I} = 2\sqrt{\beta_E^2 + \alpha^2/4} \geq 2\beta_E \tag{23.2}$$

From this relation we can estimate an upper bound

$$M \leq \delta^2 d(ES)d(EL)/16Gd(LS) \tag{23.3}$$

For instance if $d(EL) = d(LS) = 100\text{Mpc}$, $d(ES) = 200\text{Mpc}$ and the two images at the opposite radii of the lens are separated by $\beta_+ - \beta_- = 1''$ then $M \leq 6 \times 10^9$ Solar masses.

In the case where L lies on the straight line between the source and the earth, we have cylindrical symmetry. So the two images come to build a ring around the lens. Then setting in Eq. 23.2 $\alpha = 0$ we get

$$\mathrm{I}\beta_+ - \beta_-\mathrm{I} = 2\beta_E \tag{23.4}$$

From the equation

$$\beta_E^2 = 4MGd(LS)/d(ES)d(EL)$$

we can find the value M

$$M = \beta_E^2 d(ES)d(EL)/\, 4Gd(LS).$$

24. SUN RADIATION PROCESSES

The sun as the center of our system is the most important star for us. Its mass is $M \odot = 1.99 \times 10^{30}$kg, its radius $R_S = 1.4 \times 10^6$km, its luminosity $L = 2.4 \times 10^6 \text{MeV.sec}^{-1}$ and its age is about 14 Gyrs. His rotation period is 25.4 days and the temperature goes from inside to the outer surface from 7000 K to 4000 K, (K. Stumpff) .

Aside from its precious radiation, it is the place where we can test experiments for more distant stars.

The emitted radiation comes from transforming its hydrogen to He in two ways:

24.1. The proton-proton process

This process is the most probable source of energy in stars with mass similar to the Sun. The first steps are common:

p-p reaction: $H_1^1 + H_1^1 \rightarrow H_1^2 + e^+ + n + 1.74$ MeV/Mol .

Pep reaction: $H_1^1 + e^- \rightarrow n, H_1^1 + n \rightarrow D_1^2 + 1.44$ MeV/Mol .

$$D_1^2 + H_1^1 \rightarrow He_2^3 + \gamma$$

The second reaction: $He_2^3 + He_2^3 \rightarrow He_2^4 + 2H_1^1$.

The third reaction: $He_2^3 + He_2^4 \rightarrow Be_4^7 + \gamma$

$Be_4^7 + e^- \rightarrow Li_3^7 + n + 0.86$ MeV/Mol, (with propability 0.9) and

$Be_4^7 + e^- \rightarrow Li_3^{7*} + n + 0.38$ MeV/Mol, (with propability 0.1)

$Li_3^7 + H_1^1 \rightarrow 2He_2^4 \rightarrow Be_4^7 + \gamma$

$Be_4^7 + H_1^1 \rightarrow B_5^8 + \gamma$

$B_5^8 + e^- \rightarrow Be_4^{8*} + e^+ + n + 14.06$ MeV/Mol,

$Be_4^{8*} \rightarrow 2He_2^4$

As sum

$$2H_1^1 + 2n \rightarrow He_2^4 + 3.736 \times 10^3 \text{ MeV/Mol}, \quad (24.1)$$

(W.Finkelnburg)

24.2. The carbon cycle

As we see in the Fig.24.1 one carbon reacts with one hydrogen building a nitrogen plus a positron and so on.

The final result is He_2^4 and a C_6^{12}. Thus coal took part in the reaction as catalyst.

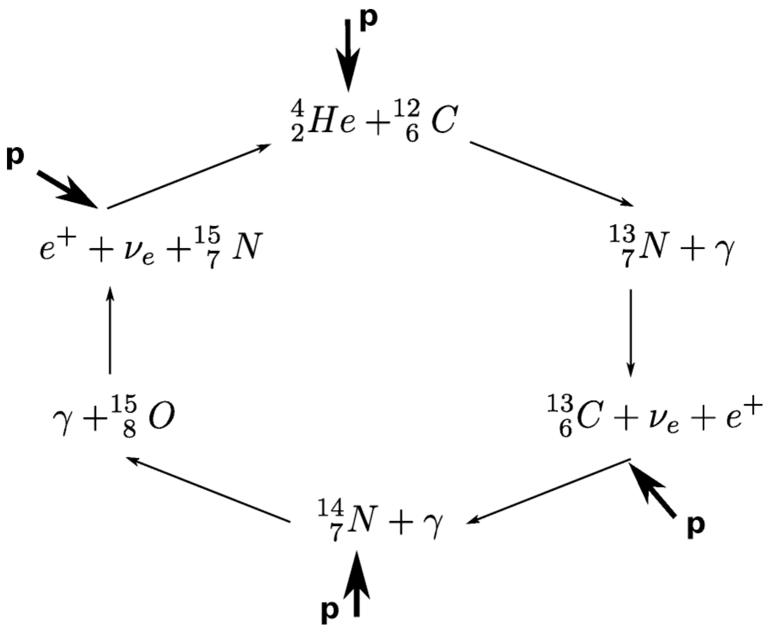

Fig.24.1. The carbon cycle.

All in all, we have once more the reaction of Eq. 24.1.

24.3. The energy through nuclear fusion and fission

Accurate measurement of the mass of the baryons of different atoms shows that it depends on the atom in which they participate.As example consider the He_2^4 nuclei with atomic mass 4.0015. It is built from 2 protons with atomic mass 1.0073 and 2 neutrons with atomic mass 1.0086 in the sum 4.0318. TheHe_2^4 is 0.0303 atomic masses lighter than the sum of his ingredients. This mass difference after fusion of two H-atoms and two neutrons to build an He_2^4 atom is transformed in energy E according to the Einstein equation

$$E = mc^2.$$

Similar argumentation holds for the nuclear plants, where f.i. U^{235} is split in Barium and Krypton under release of energy transformed in Heat and then in electric energy. 1kg U^{235} gives the same energy as 30000 kg carbon. The bindings energy for protons and neutrons in Mega electron Volts (MeV) is shown in Fig. 24.2.

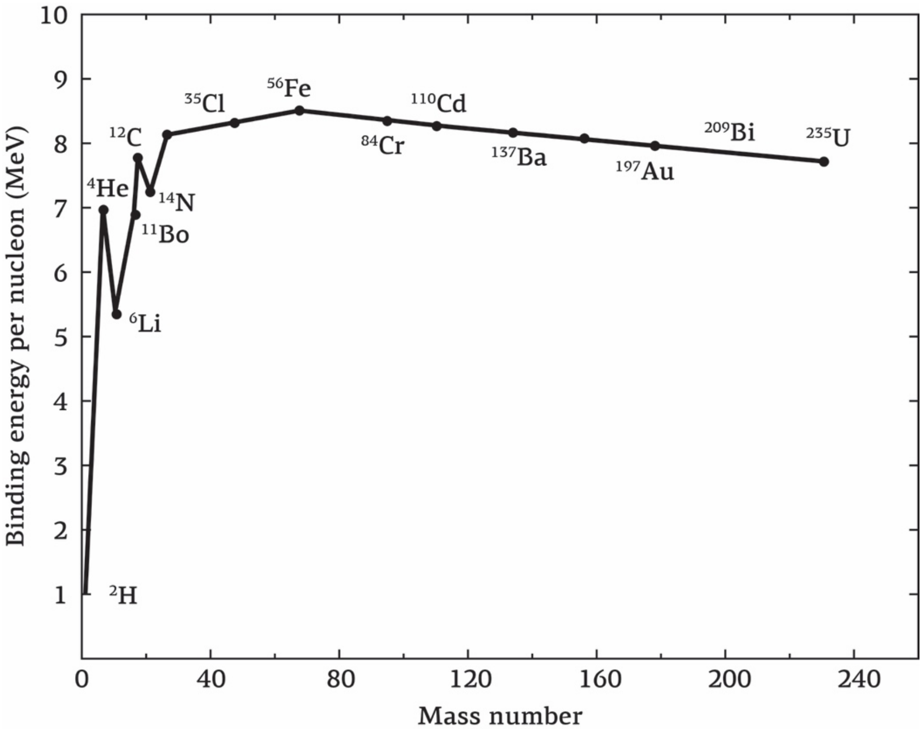

Fig.24.2

The emissions of particles $(\gamma, e^-, e, \nu, \alpha)$ from the nucleus represent the long range weak interaction while the Yukawa potential

$$V(r) = -g^2 e^{-mr}/4\pi$$

of the nucleus constituents (quarks which are bound with gluons) represent the short range strong interaction.

24.4. The Sun activity

The Sun's magnetic field changes polarity approximately every 11 years.It happens at the peak of each solar cycle as the sun's inner magnetic situation re-organizes itself. The coming reversal is before us. The poles are the herald of change. Just as Earth scientists watch our planet's polar regions for signs of climate change, solar physicists do the same thing for the sun.

The polar magnetic field of the sun weakens, goes to zero and then reappears with the opposite polarity. This is a regular part of the solar cycle.

Following T. Phillips a reversal of the sun's magnetic field is, literally a big event. The domain of the sun's magnetic influence (also known as "heliosphere") stretches billions of kilometers beyond Pluto. Changes of the field's polarity ripple all the way out to Voyager probes, on the doorstep of interstellar space.

When solar physicists talk about solar field reversals, their conversation often focuses on the "current sheet". The current sheet is a sprawling surface jutting outward from the sun's equator where the sun's slowly rotating field induces an electrical current. The current itself is small, but there is a lot of it. The amperage flows in an area 10,000 km thick and billions of km wide. Electrically speaking, the entire heliosphere is organized around this enormous sheet.

During field reversals, the current sheet becomes very wavy, the undulations like the seams on a baseball. As Earth orbits the sun, we dip in and out of the current sheet.

Transitions from one side to another can stir up stormy space weather around our planet.

Cosmic rays are also affected. These are high energy particles accelerated to nearly light speed by Supernova explosions and other violent events in the galaxy. Cosmic rays are a danger to astronauts and tele communications satellites. The current sheet acts as a barrier to cosmic rays, deflecting them as they attempt to penetrate the inner solar system. A wavy, crinkly sheet acts as a better shield against these energetic particles from deep space.

As the field reversal approaches, data show that the sun's two hemispheres are out of synchronisation.

The sun's north pole has already changed sign, while the south pole is racing to catch up. Soon however both poles will be reversed and the second half of solar max will be underway.

25. UNIT SYSTEMS

25.1. The international system (SI)

The International System of units is based on Meter-Kilogram-Second system (MKS) of units. These are the fundamental units of the Mechanic. All other units (energy, momentum, velocity, energy density and so on) can be expressed as powers of the base units. The SI-system is mostly used by engineers.

Physicists in special domains, where one or more constants are frequently used, as f.i. the velocity of light or the gravitational constant G, prefer to be released of them, setting these equal to 1 .
So we come to special unit systems.

25.2 Natural units

Natural units are used almost exclusively in cosmology and general relativity (A.L. Myers). There are often used the constants of the

velocity of light	$c = 2.9979 \times 10^{8} \mathrm{m/s}$
the reduced Planck constant	$\hbar = 1.0546 \times 10^{-34} \mathrm{J\ s}$
the Boltzmann constant	$k_B = 1.3806 \times 10^{-23} \mathrm{J\ K^{-1}}$
and the electric constant	$\varepsilon_0 = 8.8542 \times 10^{-12} \mathrm{A^2 s^4 Kg^{-1} m^{-3}}$.

So they all are set equal to 1.

Table 1 provides conversion factors for some of the variables encountered in cosmology.

Variable	SI Unit	Natural Unit	Factor	Natural unit → SI unit
mass	kg	E	c^{-2}	1 GeV $\to 1.7827 \times 10^{-27}$ kg
length	m	E^{-1}	$\hbar c$	1 GeV$^{-1} \to 1.9733 \times 10^{-16}$ m
time	s	E^{-1}	$\hbar$	1 GeV$^{-1} \to 6.5823 \times 10^{-25}$ s
energy	kg m^2 s^{-2}	E	1	1 GeV $\to 1.6022 \times 10^{-10}$ J
momentum	kg m s^{-1}	E	c^{-1}	1 GeV $\to 5.3444 \times 10^{-19}$ kg m s^{-1}
velocity	m s^{-1}	dimensionless	c	1 $\to 2.9979 \times 10^{8}$ m s^{-1}
angular momentum	kg m^2 s^{-1}	dimensionless	$\hbar$	1 $\to 1.0546 \times 10^{-34}$ J s
area	m^2	E^{-2}	$(\hbar c)^2$	1 GeV$^{-2} \to 3.8938 \times 10^{-32}$ m^2
force	kg m s^{-2}	E^2	$(\hbar c)^{-1}$	1 GeV$^2 \to 8.1194 \times 10^{5}$ N
energy density	kg m^{-1} s^{-2}	E^4	$(\hbar c)^{-3}$	1 GeV$^4 \to 2.0852 \times 10^{37}$ J m^{-3}
charge	C = A·s	dimensionless	1	1 $\to 5.2909 \times 10^{-19}$ C

Table 1. Natural units

As example we discuss the Einstein equation

$$R^{\mu\nu} - g^{\mu\nu}R/2 - \Lambda g^{\mu\nu}=8\pi G T^{\mu\nu}.$$

The Planck mass is

$$m_P = \sqrt{\hbar c/G} = 2.1764 \times 10^{-8}\mathrm{Kg}.$$

In natural units $m_P = 1.2209 \times 10^{19}\mathrm{GeV}$. Replacement of the gravitational constant in Einstein’s equation with the Planck mass gives

$$R^{\mu\nu} - g^{\mu\nu}R/2 - \Lambda g^{\mu\nu} = 8\pi T^{\mu\nu}/m_P^2.$$

Continuing with natural units, the energy momentum tensor has units of energy density GeV^4 and the Planck mass units of GeV. The RHS of the equation therefore has units of GeV^2. On the LHS of the equation, the metric tensor, $g^{\mu\nu}$ is dimensionless so the Ricci tensor $R^{\mu\nu}$, Ricci scalar R and the cosmological constant Λ all have natural units of GeV^2, or mass squared since mass and energy are equivalent.

Let us consider the cosmological constant Λ which has the same units as the energy momentum tensor $T^{\mu\nu}$ and called the energy density of the vacuum ρ_V:

$$\rho_{EV} \approx 3 \times 10^{-47} \mathrm{GeV}^4.$$

The mass density of the vacuum is $\rho_{MV} \approx 0.92 \times 10^{-26} \mathrm{Kg/m^3}$.

$$\Lambda = 8\pi\rho_{EV}/m_P^2 = 5.06 \times 10^{-84} \mathrm{GeV}^2.$$

This natural unit converted to the SI unit gives

$$\Lambda = 1.3 \times 10^{-52} \mathrm{m}^{-2}.$$

Conversion of the vacuum energy density from natural units in SI units gives

$$\rho_{EV}=6.3 \times 10^{-10} \mathrm{J\ m}^{-3}.$$

25.3. Geometrized units

In gravitational problems appear often the constant

Speed of light $c = 2.9979 \times 10^8 \mathrm{m/s}$, and the

Newton's constant $G = 6.6743 \times 10^{-11} \mathrm{m^3 kg^{-1} s^{-2}}$.

So it is comfortable to take them

$$c = G = 1$$

and the result is the conversions Table 2.

Variable	SI Unit	Geom. Unit	Factor	Geometrized unit → SI unit
mass	kg	m	c^2G^{-1}	1 m $\to 1.3466 \times 10^{27}$ kg
length	m	m	1	1 m $\to$ 1 m
time	s	m	c^{-1}	1 m $\to 3.3356 \times 10^{-9}$ s
energy	kg m^2 s^{-2}	m	c^4G^{-1}	1 m $\to 1.2102 \times 10^{44}$ kg m^2 s^{-2}
momentum	kg m s^{-1}	m	c^3G^{-1}	1 m $\to 4.0370 \times 10^{35}$ kg m s^{-1}
velocity	m s^{-1}	dimensionless	c	1 $\to 2.9979 \times 10^8$ m s^{-1}
angular momentum	kg m^2 s^{-1}	m^2	c^3G^{-1}	1 m^2 $\to 4.037 \times 10^{35}$ kg m^2 s^{-1}
force	kg m s^{-2}	dimensionless	c^4G^{-1}	1 $\to 1.2102 \times 10^{44}$ kg m s^{-2}
acceleration	m s^{-2}	m^{-1}	c^2	1 m^{-1} $\to 8.9875 \times 10^{16}$ m s^{-2}
energy density	kg m^{-1} s^{-2}	m^{-2}	c^4G^{-1}	1 m^{-2} $\to 1.2102 \times 10^{44}$ kg m^{-1} s^{-2}

Table 2. Geometrized Units

25.4. Units for Special relativity

In the Special Theory of Relativity the constant speed of light $c = 2.9979 \times 10^8 m/s$ is often used. Setting it $c = 1$,we become the Table 3.

Variable	SI Unit	SR Unit	Factor	SR unit → SI unit	
mass	kg	kg	1	1 kg	$\rightarrow$ 1 kg
length	m	m	1	1 m	$\rightarrow$ 1 m
time	s	m	c^{-1}	1 m	$\rightarrow 3.3356 \times 10^{-9}$ s
energy	kg m^2 s^{-2}	kg	c^2	1 kg	$\rightarrow 8.9875 \times 10^{16}$ kg m^2 s^{-2}
momentum	kg m s^{-1}	kg	c	1 kg	$\rightarrow 2.9979 \times 10^8$ kg m s^{-1}
velocity	m s^{-1}	dimensionless	c	1	$\rightarrow 2.9979 \times 10^8$ m s^{-1}
angular momentum	kg m^2 s^{-1}	kg m	c	1 kg m	$\rightarrow 2.9979 \times 10^8$ kg m^2 s^{-1}
force	kg m s^{-2}	kg m^{-1}	c^2	1 kg m^{-1}	$\rightarrow 8.9875 \times 10^{16}$ kg m s^{-2}
acceleration	m s^{-2}	m^{-1}	c^2	1 m^{-1}	$\rightarrow 8.9875 \times 10^{16}$ m s^{-2}
energy density	kg m^{-1} s^{-2}	kg m^{-3}	c^2	1 kg m^{-3}	$\rightarrow 8.9875 \times 10^{16}$ kg m^{-1} s^{-2}

Table 3. Special Relativity Units

Appendix 1

Quantity	Symbol	Value unit
Speed of light in vacuum	c	2.997925×10^{8} m s^{-1}
Planck constant	h	6.626069×10^{-34} J s
Planck constant, reduced	$\hbar$	1.054572×10^{-34} J s
		$= 6.582119 \times 10^{-22}$ MeV s
Conversion constant	$\hbar c$	197.326963 MeV fm[a]
Electron charge magnitude	e	1.602176×10^{-19} C
Electron mass	m_e	0.510999 MeV/c^2
Proton mass	m_p	938.272013 MeV/c^2
		$= 1.672622 \times 10^{-27}$ kg
Neutron mass	m_n	939.56536 MeV/c^2
Avogadro constant	N_A	6.022142 mol^{-1}
Boltzmann constant	k	1.380650×10^{-23} J K^{-1}
		$= 8.617343 \times 10^{-5}$ eV K^{-1}
Fine structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	7.297353×10^{-3}
		$= 1/137.036000$[b]
Classical electron radius	$r_e = \alpha\hbar/m_e c$	2.817940×10^{-15} m
Bohr radius	$a_\infty = (1/\alpha)\hbar/m_e c$	0.529177×10^{-10} m
Thomson cross section	$\sigma_T = 8\pi r_e^2/3$	0.665245 barn[c]
Gravitational constant	G_N	6.67428×10^{-11} m^3 kg^{-1} s^{-2}
		$= 6.70881 \times 10^{-39}\hbar c$ (GeV/c^2)$^{-2}$
Planck mass	$\sqrt{\hbar c/G_N}$	1.22089×10^{19} GeV/c^2
Planck length	$\sqrt{\hbar G_N/c^3}$	1.61624×10^{-35} m
Planck time	$\sqrt{\hbar G_N/c^5}$	5.39124×10^{-44} s
Fermi coupling constant	$G_F/(\hbar c)^3$	1.16637×10^{-5} GeV^{-2}
$W^{\pm}$ boson mass	m_W	80.398 GeV/c^2
Z^0 boson mass	m_Z	91.1876 GeV/c^2
Weak mixing angle	$\sin^2\theta_W$ [d]	0.2231
Strong coupling constant	$\alpha_s(m_Z)$	0.1176

a fm $= 10^{-15}$ m.
b At $Q^2 = 0$. $\sim 1/128$ at $Q^2 = m_W^2$.
c barn $= 10^{-28}$ m^2.
d defined by $1 - m_W^2/m_Z^2$.

Appendix 2

Numerical Constants

$\pi = 3.1415927$	$1'' = 4.84814 \times 10^{-6}$ radians
$e = 2.7182818$	$\ln 10 = 2.3025851$
$\gamma = 0.5772157$	$\zeta(3) = 1.2020569$

Physical Constants[1]

Speed of light in vacuum	$c \equiv 2.99792458 \times 10^{10}$ cm sec^{-1}
Planck constant	$h = 6.6260693(11) \times 10^{-27}$ erg-sec
Reduced Planck constant	$\hbar \equiv h/2\pi = 1.05457168(18) \times 10^{-27}$ erg sec
	$= 6.58211915(56) \times 10^{-22}$ MeV sec
Electronic charge (unrat.)	$e = 4.80320441(41) \times 10^{-10}$ esu
Electron volt	1 eV$= 1.60217653(14) \times 10^{-12}$ erg
	$\hbar c = 197.326968(17) \times 10^{-13}$ MeV cm
Fine structure constant	$\alpha \equiv e^2/\hbar c = 1/137.03599911(46)$
Electron mass	$m_e = 9.1093826(16) \times 10^{-28}$ g
	$m_e c^2 = 0.510998918(44)$ MeV
Rydberg energy	$hcR \equiv m_e e^4/2\hbar^2 = 13.6056923(12)$ eV
Thomson cross section	$\sigma_T = 8\pi e^4/3m_e^2 c^4 = 0.665245873(13) \times 10^{-24}$ cm^2
Proton mass	$m_p = 1.67262171(29) \times 10^{-24}$ g
	$m_p c^2 = 938.272029(80)$ MeV
Neutron mass	$m_n c^2 = 939.565360(81)$ MeV
Deuteron mass	$m_d c^2 = 1875.61282(16)$ MeV
Atomic mass unit	$m(\mathrm{C}^{12})/12 = 1.66053886(28) \times 10^{-24}$ g
	$m(\mathrm{C}^{12})c^2/12 = 931.494043(80)$ MeV
Avogadro's number	$N_A = 6.0221415(10) \times 10^{23}$/mole
Boltzmann constant	$k_B = 1.3806505(24) \times 10^{-16}$ erg/K
	$= 8.617343(15) \times 10^{-5}$ eV/K
Radiation energy constant	$a_B = \frac{8\pi^5 k_B^4}{15h^3c^3} = 7.56577(5) \times 10^{-15}$ erg cm^{-3} K^{-4}

[1] From *Review of Particle Physics*, S. Eidelman *et al.* (Particle Data Group), *Phys. Lett. B* **592**, 1 (2004).

Appendix 3

Astronomical Constants[2]

Julian year	1 year $\equiv$ 365.25 days $= 3.1557600 \times 10^7$ sec
Light year	1 light (Julian) year $= 9.460730472 \times 10^{17}$ cm
Mean earth-sun distance	1 A.U. $= 1.4959787066 \times 10^{13}$ cm
Parsec	1 pc $\equiv 648000/\pi$ A.U. $= 3.0856776 \times 10^{18}$ cm $= 3.2615638$ light (Julian) year
Solar mass	$M_\odot = 1.9891 \times 10^{33}$ g
Solar luminosity	$L_\odot = 3.845(8) \times 10^{33}$ erg sec^{-1}

Apparent luminosity for apparent magnitude m

$$\ell = 2.52 \times 10^{-5} \text{ erg cm}^{-2} \text{ sec}^{-1} \times 10^{-2m/5}$$

Absolute luminosity for absolute magnitude M

$$\mathcal{L} = 3.02 \times 10^{35} \text{ erg sec}^{-1} \times 10^{-2M/5}$$

For a Hubble constant $H_0 = h \times 100$ km sec^{-1} Mpc^{-1}:

Hubble time	$H_0^{-1} = 3.0857\, h^{-1} \times 10^{17}$ sec $= 9.778\, h^{-1} \times 10^9$ years
Hubble distance	$c/H_0 = 2997.92458\, h^{-1}$ Mpc
Critical density	$\rho_{\text{crit}} \equiv \frac{3H_0^2}{8\pi G} = 1.878\, h^2 \times 10^{-29}$ g cm^{-3} $= [0.00300 \text{ eV}]^4\, h^2$

[2]From *Allen's Astrophysical Quantities*, ed. A. N. Cox (AIP Press, New York, 2000).

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Starting from collision of two bodies in the three-body problem, the question arose to continue this process to more massive bodies and reach the margin of the Big-Bang. In this route, I felt the need to make the different meanings of the Cosmology more accessible. I tried to give a physical explanation of some strange events in the universe.

From the contents

- Periodic collision orbits in the three-body problem
- Birth of the satellites
- A model for the Big-Bang
- The interpretation of the negative pressure and the inflation
- The evolution of the universe
- The early vacuum epoch
- Gauge theories
- The Higgs mechanism and the Higgs boson
- The inflation theory
- The radiation epoch
- The matter epoch
- The processes during the temperature fall
- The time-varying vacuum energy density (Quintessence)
- The star evolution
- Ages of the universe calculated and measured
- Interaction of the photon with bright and dark matter

Pantelis Delibaltas studied Physics at the University of Munich and obtained his diploma in the Technical University of Munich. He obtained his Ph.D. degree for periodic orbits in the three-body problem at the Aristotle University of Thessaloniki. In a short time, he published a study for collisions in the same problem. He has been a Professor at the Alexander Technological Educational Institute of Thessaloniki.